Is There a Gravitational Effect Which Is Analogous to Electrodynamic Induction?

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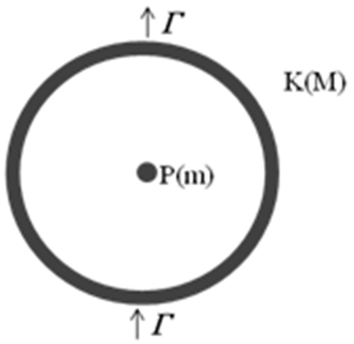
Taking a plain special case as a model, one can formulate the question raised in the title as follows. Consider a system of ponderable masses consisting of a spherical shell *K* with mass *M*, which is homogeneously distributed over the surface of the sphere, and the material point *P* with mass *m*, which is set in the center of this sphere. Does a force act on the fixed material point *P* if I impart an acceleration Γ to the shell *K*? The following arguments will induce us to view such a force effect as really being present and will give us its magnitude in first approximation.



1. According to the theory of relativity, the inertial mass of a closed physical system depends on its energy content in such a way that an increase of the energy of the system by *E* will increase the inertial mass by $\frac{E}{c^2}$, where *c* denotes the velocity of light in a vacuum. Thus, if *M* denotes the inertial mass of *K* in the absence of *P*, and *m* denotes the inertial mass of *P* in the absence of *K*, or, in other words, if $M + m$ denotes the inertial mass of the system consisting of *P* and *K* together in case where *m* is infinitely far from *K*, then it follows that the inertial mass of the system consisting of *K* and *m* possesses the value

$$M + m - \frac{kMm}{Rc_2} \tag{1}$$

if *m* is in the center of *K*, where *k* denotes the gravitational constant and *R* the radius of *K*. For $\frac{kMm}{R}$ (at least in first approximation) is the energy that one must apply in order to transport *P* from the center of *K* to infinity.

2. In a paper that will shortly appear in the *Annalen der Physik*, I have shown, based on a hypothesis about the nature of the static gravitation field, that

a material point moves in a static gravitational field according to the following equations:

$$\frac{d}{dt}\left\{\frac{\frac{\dot{x}}{c}}{\sqrt{1-\frac{q^2}{c^2}}}\right\}=\frac{-\frac{dc}{dx}}{\sqrt{1-\frac{q^2}{c^2}}}+\frac{R_x}{m}, etc.$$

Here, $\dot{x}=\frac{dx}{dt}$, and q denotes the velocity of the material point, m its mass, R_x the force acting on it, c the velocity of light, which is to be viewed as a function of the coordinates x, y, z. From these equations it follows, among other things, that $\frac{mc}{\sqrt{1-\frac{q^2}{c^2}}}$ is to be regarded as the energy of the material point, and $\frac{m}{2}\frac{q^2}{c}$, in first approximation, as its kinetic energy. In order to obtain the kinetic energy in the customary unit, one has to multiply this expression by the constant c_0, which is equal to the velocity of light at infinity; let the latter be equal to the average velocity of light in our gravitational potential. Thus, in the customary units the kinetic energy L is

$$L=\frac{m}{2}q^2\frac{c_0}{c}$$

In order to know the expression for L for an arbitrary place, we still have to determine c as a function of $x\ y\ z$. In accordance with the indicated equation of motion, for a sufficiently slowly moving point subjected to no forces aside from the gravitation field,

$$\ddot{x}=-c\frac{dc}{dx}, etc.,$$

Or, if one defines the gravitational potential Φ in similar manner,

$$\frac{d\Phi}{dx}=c\frac{dc}{dx}, etc.$$

After integration, this yields with sufficient accuracy, if Φ_0 denotes the gravitational potential prevailing at infinity,

$$\Phi_0-\Phi=c_0\left(c_0-c\right)=c_0^2\left(1-\frac{c}{c_0}\right)$$

$$\text{or }\ \frac{c}{c_0}=1-\frac{\Phi_0-\Phi}{c_0^2}.$$

For the material point in the interior of K, $\Phi_0 - \Phi$ is equal to $\frac{kM}{R}$, so that one obtains for it approximately

$$L_P = \frac{m}{2} q^2 \left(1 + \frac{kM}{Rc_0^2}\right),$$

and hence for an inertial mass m' influenced by K

$$m' = m + \frac{kmM}{Rc_0^2} \qquad (2)$$

The result is of great interest in itself. It shows that the presence of the inertial shell K increases the inertial mass of the material point P inside the shell. This suggests that the *entire* inertia of a mass point is an effect of the presence of all other masses, which is based on a kind of interaction with the latter.[1] The degree to which this conception is justified will become known when we will be fortunate enough to have come into possession of a serviceable dynamics of gravitation.

It is clear that, in the same way, the presence of P increases the inertial mass of K. By means of an argument totally analogous to the one just presented, one obtains for the inertial mass M' of K influenced by the presence of P,

$$M' = M + \frac{kmM}{Rc_0^2} \qquad (3)$$

3. We now seek the forces F and f necessary to impart the accelerations Γ and γ to the masses M or m in a given direction. If A, a, and α denote coefficients that are unknown for the time being, then we will have to set

$$\left.\begin{aligned} F &= A\Gamma + \alpha\gamma \\ f &= a\gamma + \alpha\Gamma \end{aligned}\right\} \qquad (4)$$

The coefficients of the second term (α) are chosen to be the same in the two equations, since the reaction of K on P when only K is accelerated must obviously be equal to the reaction of P on K when only P is accelerated.

The coefficients, A, a, and α follow from the consideration of the three special cases to which equations (1), (2) and (3) refer.

In the first case, K and P have the same acceleration. Let this common acceleration be γ. From (4) and (1) one obtains

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[1] This is exactly the same point of view that E. Mach advanced in his astute investigations on this subject. (E. Mach, “Die Entwicklung der Prinzipien der Dynamik. Zweites Kaopitel. Newtons Anischten über Zeit, Raum und ewegung.” [*The science of mechanics: a critical and historical account of its development.* Chapter 2, “Newton’s views on time, space and motion.”]

$$F+f=(A+a+2\alpha)\gamma=\left(M+m-\frac{kMm}{Rc^2}\right)\gamma$$

or

$$A+a+2\alpha=M+m-\frac{Mkm}{Rc^2} \tag{1a}$$

In the second case, in which *P* alone is accelerated, one has, according to the second of equations (4) and according to (2),

$$f=a\gamma=\left(m+\frac{kmM}{Rc^3}\right)\gamma$$

or

$$a=m+\frac{kmM}{Rc^2} \tag{2a}$$

The third case yields, analogously,

$$A=M+\frac{kmM}{Rc^2} \tag{3a}$$

From equations (1a), (2a) and (3a) we obtain

$$\alpha=-\frac{3}{2}\frac{kMm}{Rc^2}.$$

In the case where only *K* is accelerated, but *P* kept fixed, the second of equations (4) assumes the form, using the value of α that was just found:

$$(-k)=\frac{3}{2}\frac{kmM}{Rc^2}\Gamma.$$

k is here the force that must be exerted on the material point *P* in order for it to remain at rest; thus, *(-k)* is the force exerted (induced) on *P* by the spherical shell *K*, which possesses the acceleration Γ. This force has the same sign as the acceleration, in contrast to the corresponding interaction between equivalent electrical masses.