AUTHOR's NOTE 2/25/2015 – The author's continued work in the field has resulted in new arguments that a fully curved metric can be reasoned from equivalence. This affects the distance to the center of gravitational mass. Otherwise Laws of Inertia remain correct. A preliminary copy of the new paper, in peer review, can be obtained from ResearchGate or Academia.edu, title: "**Paths to Static Spacetime Curvature**."

Isotropy, equivalence and the laws of inertia

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An analysis of the appearance of time and motion in an accelerated frame gives the result expected by Einstein and others of an apparent mass increase in proportion to potential. This leads to a set of transformations we call the laws of inertia. The resulting inertia is isotropic. One can infer that these results apply to a gravitational field due to the Einstein Equivalence Principle. This removes an objection to Mach's Principle based on possible anisotropy. Further exploring the gravitational analogy reveals non-physical properties the analogy must have for an acceleration to be "equivalent" to gravity for weak field effects, such as precession and light bending. The new formulation, Modified Equivalence, clarifies the literature about what is or is not derivable from equivalence by showing exactly where the deficiency lies in ordinary equivalence, without resorting to Riemannian mathematics.

Une analyse de l'apparence du temps et du mouvement au sein d'un cadre accéléré donne lieu au résultat prédit notamment par Einstein, à savoir une augmentation de masse par rapport au potentiel. Ce phénomène entraîne une série de transformations que nous désignons sous le terme de principes de l'inertie. L'inertie qui en résulte est isotrope. Nous pouvons déduire que ces résultats s'appliquent à un champ gravitationnel en raison du Principe d'Équivalence d'Einstein. Cette observation supprime une objection au Principe de Mach découlant d'une anisotropie éventuelle. Une poursuite de l'analogie gravitationnelle révèle des propriétés non physiques que doit posséder l'analogie pour que l'accélération soit « équivalente » à la gravité dans le cadre de l'effet des champs faibles, tels que la précession et la déviation de la lumière. La nouvelle formulation, l'Équivalence modifiée, apporte des clarifications dans le corpus portant sur les éléments qui sont dérivables ou non de l'équivalence, en démontrant précisément où se situent les déficiences dans l'équivalence ordinaire, sans nécessiter de recours aux mathématiques de Riemann.

KEYWORDS

Equivalence, inertia, mass, acceleration, general relativity, gravity, precession, light bending, Mach, isotropy

I. SCOPE OF EQUIVALENCE

The Principle of Equivalence is due primarily to Einstein but rooted in the observations of Galileo, Eötvös, Dicke and others that all objects fall at the same rate. The Weak Equivalence Principle (WEP) reasons that therefore gravity must act equally on all forms of inertia (or energy). The Einstein Equivalence Principle (EEP), used as a guide in the formulation of General Relativity Theory (GRT), holds that in the neighborhood of a point in space all physical experiments will give the same result whether conducted in an accelerated frame of reference, or supported (as on a planetary surface) in a gravitational field, and that free fall is an inertial reference frame. The limitation of equivalence to the neighborhood of a point in space attempts to deal with the problem of convergence of gravitational force on an apparent point at the center of the source mass. Force will be along parallel lines in an accelerated frame of reference.

Einstein published a famous analysis in 1911 [1] in which he showed how equivalence explains light bending, but fortunately the prediction was never tested, as it gave only half the correct value. The full theory of general relativity gives the correct value. Various investigators have attempted to extend an equivalence-based analysis to derive all relativistic effects, including light bending and orbital precession. Lenz and Schiff in 1960 developed a derivation of the Schwarzschild metric based on equivalence, special relativity, and Newtonian gravity [2], in an effort to demonstrate limitations of experimental tests of general relativity. This would appear to give not only light bending but also planetary precession. However, two rebuttals soon appeared [3] [4] showing that the Schwarzschild metric was not possible from such assumptions, which we are inclined to accept based on our own analysis. In the current paper, we will use equivalence to analyze the question of the isotropy of inertia in a gravitational field, and determine conditions under which equivalence can explain more than in Einstein's 1911 paper, but not the Schwarzschild metric.

II. INERTIA AND GRAVITATION

Gottfried Wilhelm von Leibniz and Bishop George Berkeley took issue with Newton's idea of absolute space and time. Ernst Mach later elaborated these objections, saying that motion was only definable relative to other matter, and that water would crawl up the sides of a bucket if the universe rotated around it [5]. While Mach did not quantify this theory further, Einstein did in a 1912 paper [6] relating inertia to gravitational potential energy divided by c^2. In 1917, convinced that GRT incorporated Mach's Principle, Einstein said, "In a consistent theory of relativity there can be no inertia relatively to space, but only an inertia of masses relatively to one another. If, therefore, I remove a mass to a sufficient distance from all other masses in the universe, its inertia must fall to zero [7]."[1] Lense and Thirring's analysis of rotational frame-dragging was heralded as a Machian effect in 1918-21 [8]. Einstein's 1921 book defends his position in detail [9], but by 1949 he has reversed himself and laments, "…the attempt at such a solution does not fit into a consistent field theory… [7]," presumably referring to a boundary condition problem which was by then known.

In 1953 Sciama argued that the GRT formulation was inadequate, and derived inertia from Maxwell-like gravitational equations [10], which today are called linearized GRT [11]. Sciama did not produce a full GRT formulation. In 1957 Davidson answered Sciama, showing that the relations Sciama derived are also contained in GRT for reasonable boundary conditions [12]. In 1962 Brans argued the only way GRT can influence matter is through the metric, which can be transformed away for an arbitrarily small laboratory [13]. Brans and Dicke then put forward their own theory of gravity with a more explicit formulation of inertia [14].

The possibility that Machian inertia might be anisotropic was suggested by Cocconi and Salpeter in 1958 [15]. This was not a derivation, only a suggestion of the possibility that acceleration relative to local masses (such as the sun or the Milky Way) might produce an inertial reaction which was dependent on the angle between the acceleration and a radial vector from a test particle to the mass center. In 1960 Cocconi and Salpeter proposed that the Mössbauer effect provided a sufficiently sensitive mechanism for testing this hypothesis [16]. Results of several experiments are summarized by Hughes [17]. An experiment by Drever [18] limits the possible asymmetry to $\Delta m / m \leq 5\text{x}10^{-23}$.

The interpretations of these results are summarized by Horák [19], and vary widely, with Dicke and Epstein interpreting them as supporting Mach's Principle, Weber interpreting them as providing "no support," and others concluding the interpretation is still an open question. Cocconi and Salpeter themselves, as well as Hughes, consider anisotropy of inertia as a possibility only, not an inevitable consequence of Mach's Principle. Weber's view, however, was taken up by other influential authors, for example Weinberg [20]. Horák rebuts Weber's view within the framework of GRT and concludes "the substantial identity of the physical meaning of the Mach principle and that of general relativity implies their simultaneous validity."

In 1979 MacKenzie gave an analysis of an accelerated cylinder of masses which induces a small g-field via frame dragging (gravitomagnetic) effects [21]. MacKenzie finds a mass increase and other relativistic effects in proportion to potential in the induced g-field, itself a weak second order effect. The inertia that arises according to this argument is anisotropic.

Continuing the discussion in 1992, Treder "reminds"

[1] This quote from Einstein is used only to show his continuing interest in the Machian relativity of inertia. Presumably he refers to the inertia of the object relative to the now-distant mass of the rest of the universe. Later in the paper we show changes in inertia are only detectable relative to non-self objects.

colleagues that "Mach's relativity of inertia does not necessarily imply anisotropy of inertial masses [even] in an anisotropic universe… [22]." There appear to be continuing differences of opinion on this subject.

The plan of the current paper is to use an analysis of a pair of accelerating reference frames to show how a test particle is viewed by an observer who is at a different height. Effects are first order, and will significantly dwarf second order effects. This analysis will suggest an answer to the question of isotropy or anisotropy of locally induced inertia. By the Einstein Equivalence Principle we can then infer characteristics of the locally induced component of inertia from nearby masses (e.g. the sun or Milky Way). In the process, the scope and effectiveness of equivalence as a tool for understanding weak field trajectories will become better understood, via a special condition that will be introduced. But we will not be able to construct a full model of GRT or a Schwarzschild metric.

III. EQUIVALENCE AND ISOTROPY

Consider an elevator, either in free space undergoing acceleration a, or suspended in a gravitational field of strength $g = -a$. According to Einstein's equivalence principle, locally it doesn't matter which. The height of the elevator is taken to be Δh, sufficiently small that the equivalence principle is valid, and also sufficiently small that the change in velocity of the elevator over the distance Δh is much less than the speed of light c. Test particles A and B cross the elevator with identical velocity, v_x, measured locally, bound in the elevator's frame of reference (as in a track, for example). A is at the top and B the bottom, so they are Δh apart along the direction of acceleration. At identical intervals of horizontal distance, Δx, the test particles transmit a progress signal to one another, which may be interpreted as a clock, but also reflects exactly their velocity. For each particle, the time between transmissions is $\Delta t_x = \Delta x / v_x$.

The time of transit of signals Δt_y from bottom to top, given the assumption that the change in the elevator's velocity change over the interval is much less than c, is approximately $\Delta t_y = \Delta h/c$. The velocity attained by the top relative to the transmission point is $v_y = a\Delta t_y = a\Delta h/c$. The Doppler shift for all information transmitted at the speed of light from bottom to top is $z = v_y/c = a\Delta h/c^2$. The observed frequency of arrival of progress signals from B at the top then is $f_B` = f_B / (z+1)$. We adopt the symbol Γ for $z+1$, and substitute $z = a\Delta h/c^2$ as follows:

$$\Gamma = 1 + a\Delta h / c^2 \qquad (1)$$

Taking the reciprocal of the frequency shift as time dilation, we conclude that B's progress intervals are seen at the top to be longer by the factor:

$$\Delta t_x' = \Delta t_x \Gamma \qquad (2)$$

Primed quantities are referenced to the top of the elevator. Since the progress signals effectively measure velocity, then the velocity v_x appears slowed:

$$v_x' = v_x / \Gamma \qquad (3)$$

We make no assumptions about the magnitude of v_x. It may be anything, up to and including the velocity of light. But since $v_y << c$ we have a simple correspondence between the observations at the top and bottom of the elevator. An observer at the bottom will see events moving more quickly in the top's frame, in the inverse of (2) and (3).

We may think of viewing events through a Doppler shift, or time dilation, as like viewing a movie in slow motion. The transforms we will develop are like writing equations of motion for the slow motion movie, which obey the laws of physics in the time scale of the movie. Now we address the twin problems of mass and momentum. Only one of them can be conserved in this situation.

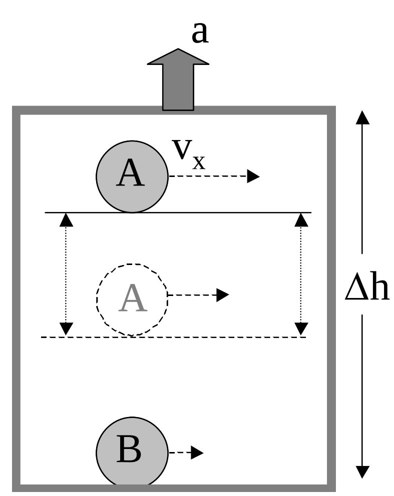

Fig. 1 Apparent velocity due to Doppler shift

in which an object free falls but does not change height.

Figure 1 shows the previously described thought experiment, with velocities diagrammed from the point of view of the top. Object A is on a frictionless track which may be lowered and raised. The velocity of A will appear to vary with height due to the Doppler Effect. Momentum is a vector and there appear to be no lateral forces acting which could account for a change in momentum. But potential energy is varying with height in exactly the same proportion that A's mass would have to vary in order to explain the velocity effect. Therefore it seems reasonable to assume momentum is conserved, and mass, as remotely observed, is the quantity that varies. This gives:

$$\mathbf{p}' = \mathbf{p} \tag{4}$$

$$m' = m\Gamma \tag{5}$$

Of course, one might argue the mass is not really increased. This is an artifact of measurement through an unavoidable Doppler shift that applies to all information flowing between the bottom and top. An observer at the top has a choice, either to manipulate measurements so as to restore the values seen by a local observer at the bottom and compute with those values. Or the observer at the top can take measurements at face value in his local reference frame, in which case the mass increase must be accepted.

The mass increase of (5) may be interpreted as arising from the potential $a\Delta h$, which in the gravitational field of a source mass M at radius R corresponds to GM/R. This in turn can be interpreted as the Machian inertia induced by the local potential field of M [6] [10] [12] [23] [24] [25]. This is the inertia that was thought possibly to be anisotropic [15] [16] [17] [18] [19] [20]. Our derivation was, however, done using lateral momentum, perpendicular to the radius to any central mass M, and is based only on potential (by way of the Doppler shift), so it is likely to be isotropic, but we must show that (5) holds for arbitrary motion.

Acceleration (or gravity) severely distorts vertically aligned motion components, and accelerated clocks do not work well. Instead, clocks used to look for anisotropic effects in inertia, such as the Mössbauer effect, have been free falling in space, not accelerating. In an equivalence setup, there is no analog of an orbit in which an object free falls but does not change height.

The best we can do is use an inertial free falling clock. This will have a velocity Doppler Effect, but if we design the device so that there is only one measurement point for progress signals, we can perform identical experiments with the clock in different orientations, and in the local inertial frame we will get identical results. If the measurement point is initially co-moving with the bottom of the elevator at the start of the experiment, identical progress signals will be transmitted regardless of clock orientation.

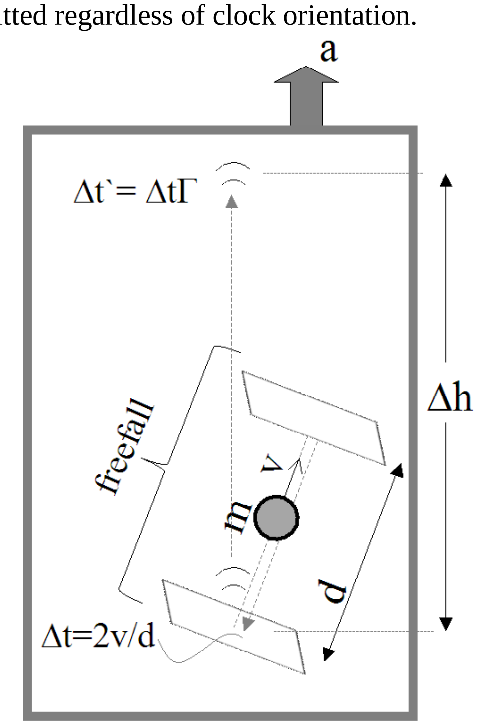

Fig 2 Free falling velocity clock

Figure 2 shows a simple velocity clock (an object bouncing between two plates). The progress signal originates only on one of the plates. The orientation of this clock will not affect the measurement, so the mass *m* cannot depend on the direction of motion.

Therefore, the appearance of inertia change with acceleration potential is isotropic. If the Einstein Equivalence Principle is correct, we can apply these results to gravity by substituting the field of a mass M, at sufficient radius R that the size of the elevator does not become a factor, and we can approximate the acceleration in the elevator as $a = -g = -GM/R^2$. As a philosophical guide, which is the way Einstein used his equivalence principle, this suggests that the apparent mass changes over larger intervals are also isotropic. Thus we claim equivalence suggests that local inertia is isotropic.

IV. LAWS OF INERTIA

The reference frame transformations (2) through (5) imply additional transformations which will be needed for trajectory analysis. Each of these will be discussed:

$$\mathbf{F}' = \mathbf{F} / \Gamma \quad (6)$$

$$\mathbf{A}' = \mathbf{A} / \Gamma^2 \quad (7)$$

$$L' = L \quad (8)$$

$$E' = E / \Gamma \quad (9)$$

The author calls these the "laws of inertia" because they were discovered to be consequences of changes in inertia. However, we have deduced them from equivalence, by way of time dilation, so they could just as well be called the laws of equivalence, or the laws of time, but as the former doesn't connote very much, and the latter seems rather overreaching, the laws of inertia by comparison seems a useful name.

These transformation laws are similar to the transformations of special relativity, with two notable exceptions. First, the inverse transforms are not identical, they are opposite. A lower observer sees higher observers blue shifted and time accelerated, where as in special relativity all objects in different states of motion are seen time dilated. Second, without relativistic motion there is no length contraction (at least to first order), and without length contraction, part of the Lorentz time transformation is missing, the part which relates time to distance along the contracted axis of motion. An observer at the top of the elevator sees clocks at the bottom of the elevator as synchronized. Even clocks at other levels may be synchronized at some arbitrary moment of time by careful signaling and planning, and all observers will agree on the synchrony, but of course they will drift apart due to running at different rates. By contrast, observers will never be able to agree on even a moment of synchrony between multiple points in relativistically moving reference frames. The two sets of laws are so closely related that if one imagines a relativistic reference frame without length contraction (a hypothetical entity which the author calls a purely inertial frame (PIF), referring to inertia being the primary transformed quantity), the laws of inertia can be used to describe motion in that frame. To get the observed local motion, simply apply the inverse transforms. To get the relativistically observed motion, apply length contraction and the length-related component of time transformation. We will not show this in detail. The discussion is just to help understand that the laws of inertia are not entirely new, and how they fit with the reader's experience.

The force transformation (6) is perhaps the least obvious from experience. To derive this transformation we will discretize force as follows. Referring to the same elevator and test particles, let A be accelerated laterally by a series of differentially small momentum impulses that average to a certain force, $F = ma_x$, where m is the mass of A and a_x is a lateral acceleration. Let an identical series of lateral momentum impulses be transferred from the top to bottom of the elevator by the method of Figure 1. The rate of arrival will be blue shifted by Γ, which is the inverse of (6) as expected.

What about electromagnetic forces, and the conservation of charge? For forces between particles at rest or in continuous motion with respect to each other, consider that electromagnetic forces are held to be momentum impulse forces arising from quantum field interactions. For a pair of objects both immersed in a time dilation frame such as the bottom of the elevator, impulse rates for both of them would be viewed as lower by an observer higher in the frame, and thus the *apparent* force between them would conform to (6).

For forces between particles at different points in an acceleration potential, such as the top and bottom of the elevator, while the distance between these points may be constant, the particles are both accelerated, and this will distort the field between them due to the Doppler effects. However, to understand and address this we must first discuss the illusion of progressive lag and how this is handled, which is in the next section.

The transform of acceleration seems to have been first noticed by G. Ascoli (unpublished by Ascoli, but discussed and attributed in Sard [26]). It is easy to understand in terms of time dilation, since the units of acceleration have a Δt^2 term in the denominator.

The identity transformation of length L in any direction is a consequence of the lack of dependence of our derivations on relativistic velocities, but is subject to interpretation for reasons which will be elaborated. If an object is moving relativistically within the frame, the Lorentz transformation applies, and length contraction appears along its trajectory. Without relativistic motion, a co-moving inertial observer can be used to verify that lengths are the same between the top and bottom of the elevator. If an object D is initially co-moving with the top and is released into free fall, its velocity when it passes the bottom is $v_d = (2a\Delta h)^{0.5}$. The Lorentz factor for this velocity is

$$\gamma = 1/\sqrt{1-v_d^{\,2}/c^2} = (1-2a\Delta h/c^2)^{-1/2}$$
$$\Rightarrow \gamma \approx 1 + a\Delta h/c^2 = \Gamma \qquad (10)$$

This seems to be purely an effect of special relativity as it depends on the direction of motion of D, which may be deflected to move in any direction. In orbital situations, such an object will be moving in a non-radial direction.

The transformation of energy is a consequence of the Doppler shift. Photons transmitted from bottom to top undergo a frequency shift, which by the Planck-Einstein equation, $E = h\ f$, gives one factor of Γ reduction. Consider a free falling object of mass *m* in the equivalence setup we have been using. Let *m* have zero initial velocity when released at the top of the elevator. The kinetic energy of *m* at the bottom after it falls Δh is $ma\Delta h$, so the total energy can be written as $E = mc^2 + ma\Delta h = mc^2\Gamma$. This is the local energy at the bottom. If the object is then entirely converted to energy and transmitted from the bottom to the top, a factor of 1/Γ in energy is lost to the Doppler shift, giving back the original rest energy mc^2. Alternately one can find the energy which an observer at the top must imply by replacing *m* with *m*\` and *c* with *c*\` giving $E` = m`c`^2\Gamma = m\Gamma(c/\Gamma)^2\Gamma = mc^2$.

If an object *m* is brought close to a mass M such that *m's* inertia is noticeably increased, the energy transformation (9) exactly cancels that mass increase as far as retrievable energy is concerned, so energy is conserved. In other words, the inertial increase is with respect to motion of *m* and M relative to each other. This is similar to the entrapment of an object in the event horizon of a black hole. The object cannot be moved relative to the black hole, but the black hole can be moved. The mass of the system (M and *m*) is the sum of the component masses, transformed to the frame of a distant observer. This must also include the mass of any kinetic energy. So we have $M_{TOTAL} = M + m` + ma\Delta h = M + m/(1+a\Delta h) + a\Delta h$. Assuming $a\Delta h << 1$ and using the appropriate approximations, this gives $M_{TOTAL} = M + m - a\Delta h + a\Delta h = M + m$, so that as expected the locally relative mass increase of *m* does not show up in the total system mass.

If *m* is lowered on a tether and the potential energy is extracted, then kinetic energy is not present at the bottom and the total system mass is reduced by $ma\Delta h$, which becomes the binding energy [6], and must be added back if the object is to be retrieved.

V. ANALOG TO GRAVITATION

To draw conclusions about gravitation, a precise definition of the acceleration analog to gravitation is required. A thought experiment with a tether reveals a specific and somewhat unexpected condition for acceleration-gravitation equivalence. The condition will explain why previous attempts to produce features of the Schwarzschild metric have not been successful.

In this section, we seek conditions of "equivalence" in which a constant acceleration produces the following analog to gravitation: two objects separated by a differential height Δh will indefinitely remain separated by Δh according to *both* observers. This corresponds to a top observer on a supported platform, for example, and a bottom observer on a surface below. It will be shown that a non-physical mathematical abstraction of equivalence, which we will call Modified Equivalence, is the only way to satisfy this condition.

Let an object *m* be lowered in an accelerated elevator on a tether. By assumption, local observers at both top and bottom notice an acceleration *a* relative to freely falling objects. If the top observer applies (7), he finds the acceleration at the bottom appears to be a/Γ^2. The upper observer thus sees the bottom fall away, because its acceleration is too small by Γ^2. The apparent point of origination of signals received at the top gets progressively further behind, much as the way the apparent source of sound from an accelerating jet gets further behind the actual position of the jet, as illustrated in Figure 3. Consider what happens if the acceleration stops. As previously transmitted signals come in, the "image" of the bottom gradually catches up with the actual bottom. Clocks at the bottom and top will no longer be synchronized. Consider an inertial reference frame co-moving with the elevator at some past time. The elevator may now have gradually accumulated a relativistic velocity causing clock skew.

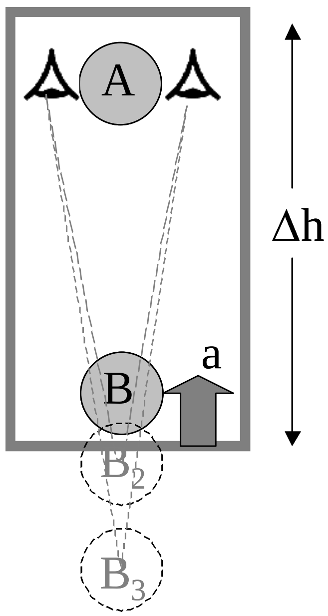

Fig. 3 Acceleration Doppler progressive lag effect

Another factor that accumulates over time is contraction of the elevator, relative to some inertial observer co-moving with the elevator at some past time. This corresponds to the accumulated clock skew.

The situation of Figure 3 is obviously not what we see when looking at an object below us in a gravitational field. We do not see objects at fixed heights appearing gradually to fall, nor do we see the distance to objects gradually shrinking over time, even over millennia. (A free object does of course fall, and a free falling observer does see lengths contract, but as discussed earlier, such an observer's observations are dependent on trajectory.)

The resolution which the author suggests is that in order to apply a finding from an equivalence setup to a gravitational field where the distance Δh is unchanging with time, we must assume that observers at the bottom of the elevator experience the acceleration which would be required for an observer at the top to "see" them at a constant distance. This means that an observer $-\Delta h$ below a top observer who sees a gravitational acceleration g` would, to satisfy the acceleration transform (7), have to experience a gravitational acceleration g of

$$g = g'\Gamma^2 \qquad (11)$$

Such an acceleration implies that object B experiences a gravitational force $F=mg\Gamma^2$. Using the force transform, the force at the top of a tether is then found to be

$$F` = F / \Gamma = mg\Gamma \qquad (12)$$

Therefore we see that under the conditions for which equivalence may reasonably be said to correspond to a gravitational field (forcing the Doppler illusion to coincide with observations in a gravitational field), a mass on a tether does indeed feel heavier. This result will directly enable the derivation of trajectories for planetary precession in weak fields. One interesting question it raises is, what is the actual potential energy if a larger than Newtonian force is involved? That question is beyond the scope of this paper.

Any suggested condition on so long established a concept as equivalence needs careful explanation and justification. Consider two alternate views of what this means.

First, the "memory" effect of acceleration will be clarified by discretizing acceleration, similar to the way force was discretized earlier. At any moment in time, consider the view of an inertial observer co-moving with the top of the elevator. Let acceleration proceed in incremental velocity steps (acceleration impulses) at the beginning of each Δt interval, followed by coasting, such that the average acceleration remains the same as before being discretized.

There are many possibilities for timing of the impulses. Consider three cases: pull, push and sync. We may imagine rockets attached to the top and bottom of the elevator which produce the impulses. The elevator does not need to have a physical structure connecting the top and bottom.

The "sync" case we will define as based on synchronized clocks during the coast phase, so that the impulses at top and bottom come at the same time in a currently co-moving inertial frame. An observer at the top will not see the bottom begin to move until a time interval $\Delta h/c$ later than its own impulse. During this time, the bottom seems to fall behind (the gap widens). From the point of view of the bottom, the top falls behind (i.e. the gap closes). This amount of change in view persists until the next round of synchronized impulses, at which time the same thing happens again, adding to the distortion of views.

In the "push" case the acceleration of the top is delayed until acceleration of the bottom is observed. In this case the bottom does not appear to lag. It appears to maintain constant distance in the accelerated view from the top. It appears to close the gap in the co-moving inertial view. The gap widens at a double rate in the view from the bottom. There is no Doppler shift or electric field distortion in the top's view.

In the "pull" view everything is reversed, with the bottom waiting for a signal from the top before accelerating.

None of these corresponds to any view in a gravitational field, in which objects at fixed heights do not appear to recede or approach each other even over eons of time, and electromagnetic fields are distorted only insofar as time dilation corresponds to Doppler shift, but the static forces from charges are not distorted, and charges do not appear to radiate.

Note that we have only hypothesized a differentially small elevator, in both height and width, in order to notice all these effects. We will show in the next section that they "make a difference" in a differentially small frame, and are able to explain weak field effects when choices are made that coincide with the observation that in a gravitational field time is dilated but the progressive illusion of a widening gap does not occur. That condition is exactly that cross-frame measurements of local physical parameters transform as we have defined them, and the field acceleration remains constant in the reference frame of any *one* observer (or in the case of a gravitational field, conforms to Newtonian acceleration in the view of any one observer).

That condition leads to the second view of what this means, which is that an observer at the top of the elevator sees objects fall at a constant rate. Though local events at lower heights slow down due to time dilation, the acceleration of falling objects does not. It has aspects of push, pull and sync equivalence, but does not seem to correspond to any physical setup. It is inspired by equivalence, with a special condition on falling acceleration, which for purposes of this paper will be referred to as a modified acceleration equivalence principle, or Modified Equivalence.

In the next two sections, we show this non-physical version of equivalence is indeed "equivalent" to weak field gravity, which implies that no physical version of equivalence can be. This is in agreement with Sacks and Ball [3], and Rindler [4]. Quoting Sacks and Ball with regard to derivations of the Schwarzschild metric from equivalence: "In all these derivations an answer of the correct form is arrived at through a combination of invalid physical interpretations and coincidences." Our present work aims at an improved understanding of equivalence by replacing "invalid interpretations and coincidences" with a single carefully reasoned deviation, based on simple observations (time dilation with the lack of a Doppler falling-away illusion in gravitational fields). In turn, the work aims at determining which aspects of equivalence may be taken with some degree of confidence to apply in gravitational fields, with specific interest in inertia.

VI. TRAJECTORY

Now we turn our attention to what kinds of trajectories can, and cannot, be inferred from Modified Equivalence. This will help confirm the role of isotropic inertia in relativistic effects. We will show that equivalence has enforced a set of transformations so that a change in inertia, or relative potential, does not in itself alter trajectory, but only time. This will guarantee that all clocks, no matter the mechanism, slow at the same rate, and that the shape of all trajectories is the same, although their timing is modified.

Consider a particle at coordinate position $\mathbf{X}$ and describe its motion according to a local observer, and a remote observer who uses a Γ transformation factor and whose measurements are noted with primes. For convenience we assume the coordinate origin and axes are superimposed such that **X`**=**X**. The equations of motion for the particle in its own frame are

$$\mathbf{v}_2 = \mathbf{v} + \mathbf{A}dt$$

$$\mathbf{X}_2 = \mathbf{X} + \mathbf{v}dt$$

The subscript "2" indicates the new position, not a selection of coordinates. In the remote observer's frame we have

$$\mathbf{v}_2\text{`} = \mathbf{v}\text{`} + \mathbf{A}\text{`}dt = \mathbf{v}/\Gamma + (\mathbf{A}/\Gamma^2)d(t\Gamma)$$

$$\Leftrightarrow \mathbf{v}_2\text{`} = (\mathbf{v} + \mathbf{A}dt)/\Gamma = \mathbf{v}_2/\Gamma$$

$$\mathbf{X}_2\text{`} = \mathbf{X}\text{`} + \mathbf{v}\text{`}dt = \mathbf{X} + (\mathbf{v}/\Gamma)d(t\Gamma)$$

$$\Leftrightarrow \mathbf{X}_2\text{`} = \mathbf{X} + \mathbf{v}dt = \mathbf{X}_2$$

Therefore the position coordinates in the trajectory will not be modified by the transforms. (If length contraction and the associated time displacement are added, these transformations can be applied to special relativity and are sufficient to explain the "fly-by principle," i.e. that a relativistic test particle passing through a solar system does not change the planetary orbits.)

If a force or acceleration does *not* transform according to the laws of inertia as we have specified, then its orbit *will deviate* from the shape of the expected Newtonian orbit. When the equivalence setup is carried over to a gravitational situation, by assumption we do not transform the gravitational acceleration. An observer who sees acceleration *a* at his own height, sees *a* for objects at other heights as well. The comparable statement for a field about a mass M is to say that an observer who sees $a = GM/R^2$ at his own height (R), sees this relation valid at all radii. We will examine the effect this has on orbits.

For a comparison baseline of gravitational effects the Schwarzschild metric will be used, which is known to give a correct result for planetary orbits in the solar system. Taking the form given by Brown [27]:

$$d^2r/d\tau^2 = -m/r^2 + \omega^2(r-3m) \qquad (13)$$

and re-writing using our notation and units, we have

$$a = -GM/R^2 + (v^2/R^2)(R - 3GM/c^2)$$

$$\Leftrightarrow a = -GM/R^2 + (v^2/R)(1-3GM/Rc^2)$$

For $3GM/Rc^2 << 1$ we can use the small *x* approximation, $1 - x \approx 1/(1+x)$, thus:

$$a = -GM/R^2 + (v^2/R)/(1+3GM/Rc^2) \qquad (14)$$

Since (14) is in the frame of the object, which is free falling, $a = 0$. What we have left is the balance of gravitational acceleration and centripetal acceleration. The Newtonian centripetal acceleration is reduced by $(1+3GM/Rc^2)$ which can be factored, ignoring high order terms, as $(1+GM/Rc^2)^3 \approx \Gamma^3$, where $\Gamma = (1+GM/Rc^2)$. We can rewrite (14) as

$$GM/R^2 \approx (v^2/R)/\Gamma^3 \qquad (15)$$

Whenever equations of orbital motion in the frame of the orbiting object can be reduced to this form, the observed value of planetary precession will be obtained.

We can derive a relation between the gravitational relativistic factor for weak fields, Γ (for $GM/Rc^2<<1$ this is equivalent to the Schwarzschild metric's time dilation factor $(1–2GM/Rc^2)^{-0.5}$), and the lateral velocity Lorentz factor $\gamma = 1/(1–v^2/c^2)^{0.5}$. For circular orbits, tangential velocity is given by:

$$v = \sqrt{GM/R} \qquad (16)$$

This is a good approximation to average velocity for near circular planetary ellipses if R is taken as the semi major axis. Substituting for *v* in the Lorentz factor formula and using the usual approximations for operations on $1 \pm x$ for $x<<1$ we have:

$$\gamma = 1/(1-GM/Rc^2)^{0.5} \approx \Gamma^{0.5} \qquad (17)$$

The total relativistic transformation factor for an orbiting mass will then be $\Gamma\gamma = \Gamma^{1.5}$.

Figure 4 shows how an accelerated frame of differential width Δx can be applied to an orbit. For simplicity, a circular orbit is assumed, which allows the orbiting object to enter and leave local accelerated frames conveniently at the same height R. In the limit as $\Delta x \rightarrow 0$ an accurate representation will be obtained.

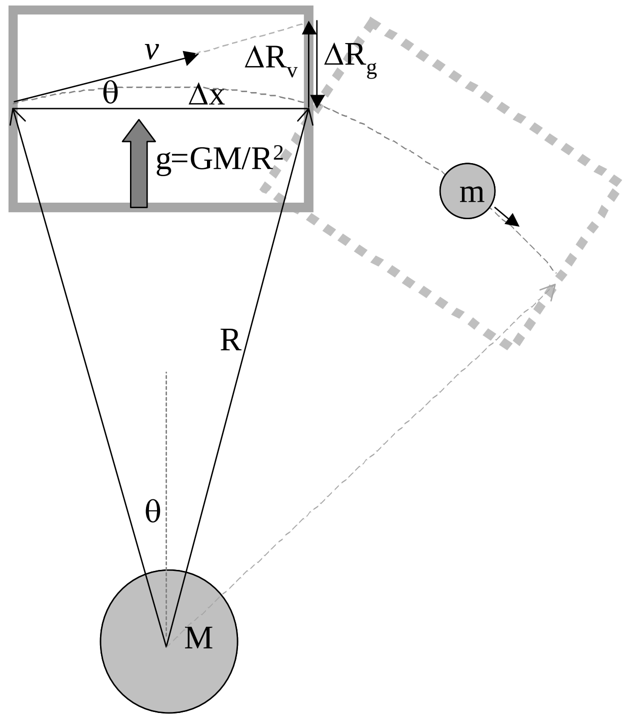

Fig. 4 Orbit represented with accelerated frames

Setting the radial displacement due to gravity ΔR_g equal to the radial displacement outward ΔR_v due to inertial continuation of v gives the expected result for balanced gravitational and centripetal force, $g = GM/R^2 = v^2/R$. This equation has been derived so far without regard to relativistic factors. Accounting for *m's* relativistic motion, notice that centripetal acceleration v^2/R doesn't change. A new Δx is marked using *m's* coordinates, leaving the diagram of the accelerated frame unchanged. The number of Δx*'s* that m finds in an orbit is not a factor since neither R nor v changes. However, the constant gravitational acceleration will be perceived through *m's* time dilation and must be transformed by the inverse of (7) giving:

$$(GM/R^2)(v\Gamma\gamma)^2 = v^2/R$$
$$\Leftrightarrow GM/R^2 = (v^2/R)/\Gamma^3 \qquad (18)$$

This has exactly the same form as our benchmark (15). We conclude that any law of gravity that produces acceleration which satisfies the Weak Equivalence Principle, and time dilation according to the relation we have given, and is otherwise Newtonian, is sufficient to explain planetary orbits. This does not, however, produce other aspects of the Schwarzschild metric, as we will see below. In any event, the conclusions from Modified Equivalence only show what trajectories are when Modified Equivalence holds. They do not constitute a theory of how gravity works. Currently all successful theories accomplish Modified Equivalence by treating gravity as geometry.

VII. LIGHT BENDING

The orbital analysis will not help with light. Einstein already investigated that falling rate [1] and found only half the observed light bending [28] [29]. But time dilation, and the consequent velocity slowing, will have a steering effect separate from any falling effect. In an acceleration setup, time dilation is only a Doppler illusion, and there is no actual bending of transverse light paths. But in a gravitational field, time dilation and velocity slowing must be taken into account.

Referring to Figure 5, consider two parts of a wave or particle separated by Δh and traveling horizontally at v and v_2 respectively.

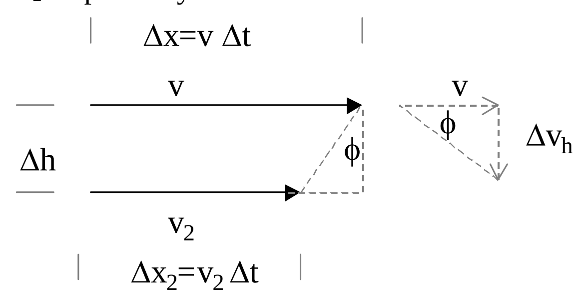

Fig. 5 Setup for speed gradient refraction

After a horizontal interval Δx we have $\Delta x = v\Delta t$, and we assume $\Delta x_2 = v_2\Delta t = (v/\Gamma)\Delta t$. Two formerly vertical points on the object will be turned at an angle ϕ such that $\tan\phi \approx \phi \approx (\Delta x - \Delta x_2)/\Delta h = (v - v/\Gamma)\Delta t/\Delta h$. The velocity vector **v** will be turned by this same angle ϕ so that a vertical velocity component Δv_h is added, where $\tan\phi \approx \phi \approx \Delta v_h/v$. Equating the two expressions for ϕ we have $\phi \approx \Delta v_h/v = (v - v/\Gamma)\Delta t/\Delta h$. We can rearrange this into an expression $\Delta v_h/\Delta t = v^2(1 - 1/\Gamma)/\Delta h$. This value $\Delta v_h/\Delta t$ is aligned with the gravitational acceleration g. Substituting for Γ using (1) and simplifying we have:

$$\frac{\Delta v_h}{\Delta t} = v^2(1-(1-g\Delta h/c^2))/\Delta h = \frac{v^2}{c^2}a \qquad (19)$$

For light, we have $v = c$ and therefore $\Delta v_h/\Delta t = g$.

Since $\Delta v_h/\Delta t$ is added to the explicit gravitational acceleration, *g*, as already noted, we have a total apparent acceleration of *2g*. Thus a double bending of light is derived for the gravitational analogy of equivalence, which agrees with observation. This finding is not dependent on what we have defined as Modified Equivalence. It is only dependent on our definition of how equivalence is used in a gravitational analogy: time dilation and velocity slowing are treated as real, not optical Doppler illusions.

Note that considering a single local frame, the downward displacement of light exiting the frame is just as Einstein gave it in 1911. What we have just seen is that the angle of the light at exit is not what would be expected from a simple dropping by that displacement. The light is turned by an additional amount due to refraction in a speed gradient. This is analogous to Huygens refraction, and results in deflections identical to Huygens refraction for light,[2] but the speed gradient may be used for objects that are not primarily described by wave motion. In fact we have generalized our computation of the deflection due to the speed gradient to apply to any object moving with any speed across the elevator. Even if the object is a point particle, the probability function of its future occurrences will be shaped by the speed gradient.

Some investigators have been puzzled at the coincidence that Huygens refraction bending should be exactly equal to Newtonian bending. From the foregoing we conclude that whenever $v = c$ this will be the case. For $v << c$ as in the case of planetary orbits, speed gradient refraction is a second order effect and usually can be ignored.

VIII. EQUIVALENCE IN GENERAL RELATIVITY

This section will compare conclusions from equivalence with results from the well known Schwarzschild metric. The proper time T in the reference frame of an object at radius R relative to the time T` measured by an observer at infinity in this metric is:

$$\begin{aligned} T &= T'(1-2GM/Rc^2)^{0.5} \\ \Leftrightarrow T' &= T/(1-2GM/Rc^2)^{0.5} \end{aligned} \qquad (20)$$

For $2GM/Rc^2 << 1$, approximations allow (20) to be rearranged as follows:

$$\begin{aligned} T` &\approx T/(1-0.5(2GM/Rc^2)) \\ \Rightarrow T` &\approx T(1+GM/Rc^2) \end{aligned} \qquad (21)$$

Expression (21) is exactly what we would expect from replacing acceleration potential $a\Delta h$ with gravitational potential *GM/R*. So in the approximation agreement is perfect. However, infinite time dilation occurs in (20) at the gravitational radius $R = 2GM/c^2$. In (21) infinite time dilation occurs only for $R = 0$. Again we find that while the achievements of Modified Equivalence are impressive regarding weak field effects, it still does not yield the Schwarzschild metric, or a noticeable property of it, an event horizon.

In an equivalence setup with a height Δh and acceleration *a*, relative velocities between the top and bottom greater than *c* are not achievable due to special relativity. This is comparable to the situation in the Schwarzschild metric where in a distant observer's frame a falling object never crosses the event horizon, but in the object's frame it can. However, in the equivalence setup, objects never become unreachable unless they are converted entirely to energy and achieve the speed of light.

We now consider the question of local inertia effects in a gravitational field. If a hypothetical idealized tether is attached from an observer to a falling object in the Schwarzschild metric, the observer would find the object slowing down, but an infinite force would be required to retrieve the object from the horizon. Due to time dilation in the object's frame, the observer would find that as the object approached the horizon, an extra force would be required to deflect the object in any direction, not just vertically. The observer could reasonably interpret this requirement as due to increased mass of the object.

[2] Huygens refraction can be formulated as entirely dependent on a ratio of velocities. Velocity of light in a medium depends on interactions which in turn depend on the wavelength of the light, but velocity of light in a gravitational field does not depend on wavelength.

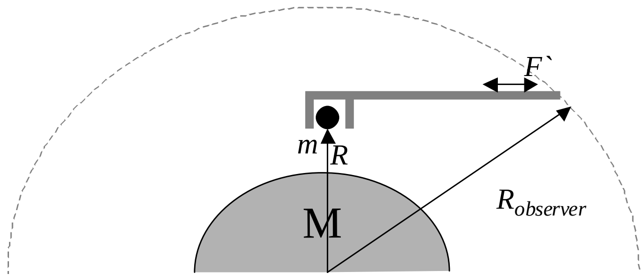

Fig. 6 Force and mass in a gravitational field

Consider the thought experiment of Figure 6, where an observer at a higher radius is equipped with a means to exert force laterally on a mass *m* at lower radius R. The laws of motion must hold for both observer and object. For convenience, suppose impulses are exerted through the apparatus, which average over time to produce a force F. Let Γ = T`/T. The object *m* will perceive the rate of arrival of either left going or right going impulses to be Γ greater than the rate the observer sends them, owing to time dilation, and will perceive the effective force to be ΓF` greater. In the object's frame, then, $F=\Gamma F`=ma$, and $a=\Gamma F`/m$, where a is lateral acceleration. Applying inertial transformations we have the acceleration perceived by the observer:

$$\begin{aligned} a' &= a/\Gamma^2 = F'/\Gamma m \\ &\Rightarrow m' = \Gamma m \end{aligned} \quad (22)$$

The observer would necessarily conclude the object's inertial mass was increased in proportion to time dilation, wherever time dilation was observed in a metric. Some readers may feel this is an interpretation, and will prefer other interpretations. The author would agree it is interpretive, but respond that the interpretation has a use in understanding Mach's Principle, experimental results on the isotropy of local inertia, and the compatibility of these ideas with general relativity.

The situation of test masses seeming to have inertia in otherwise an empty universe can possibly be understood by a limit process. Consider a series of cosmologies varied according to some parameter in which the members of the series have less and less mass. Since inertia is entirely relative, the inertia perceived in each by observers might be finite and non-zero, even as in the limit the total mass approached zero.

IX. CONCLUSION

First, there is a clear inference from equivalence that there is inertial mass increase in a gravitational field, and that this incremental increase is isotropic. It is the purpose of the current paper to suggest that most kinds of theories containing time dilation will also predict such an increase, which GRT does under reasonable boundary conditions [12]. It is also a purpose of this paper to remove any dependence on lateral gravitomagnetic effects as in [21]. Since time dilation will correspond to mass increase, inertial changes will not be observable in the frame of the mass, but only to an observer who remains at a fixed potential.

Second, Modified Equivalence and the laws of inertia can be used to explain orbital precession and light bending. But in order to maintain an analogy to gravitational observations and thus explain orbital precession, a non-Newtonian gravity is required (i.e. modified acceleration), and in any case equivalence does not seem to explain strong field effects, such as an event horizon.

In part because of the difficulties in making assertions based on Modified Equivalence, and in part because of second order effects such as those described by MacKenzie, we do not hold that the derivation of isotropy is a proof. But in light of the extremely good success of equivalence both experimentally, and as an analytical tool for understanding first order effects, it seems that it makes a convincing suggestion that inertia is isotropic.

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