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"Leading clocks lag" and the de Broglie wavelength

Robert L. Shuler, Jr.[1]

[1]*NASA Johnson Space Center, Houston, TX*
robert.l.shuler@nasa.gov or robert@mc1soft.com

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Abstract

The forgotten history of de Broglie waves as themselves artifacts of a Lorentz transform, not physical lengths and frequencies to be transformed, causes confusion for students and others. In this paper the de Broglie wavelength is derived and dependence of de Broglie frequency on velocity explained in terms of Einstein synchronized reference frame measurements of a single clock (*2-on-1*) vs. measurements of an Einstein reference frame by an observer with a single clock (*1-on-2*). A new memory aid for the relativity of simultaneity, "leading clocks lag," is introduced that puts it on a par with the heuristics "time dilation" and "length contraction" for quick and easy problem visualization.

Keywords: length contraction; simultaneity; special relativity; time dilation; de Broglie wavelength;

1. Introduction

The phrases "time dilation" and "length contraction" are easily remembered and convey an immediate sense of the character of observations involving relative motion. Einstein discusses in section 4 of his original paper on Special Relativity the "Physical Meaning of the Equations Obtained in Respect to Moving Rigid Bodies and Moving Clocks [1]," and even before that the term "contraction" was used by Lorentz [2]. But the third essential aspect of the Lorentz transform has no such shorthand expression.

In studies of student problem solving, as for example a recent report by Cormier and Steinberg [3], it is often found that students are most adept at visualizing and calculating with time dilation from the point of view of an inertial frame of reference. Length contraction is likewise an easy concept when applied to material objects, but causes confusion when no material object occupies a gap of space. Conflicting views of *simultaneity* in different reference frames even more frequently cause confusion. In this paper, for referring to the skew of clocks in another reference frame with respect to its direction of relative motion, the phrase *leading clocks lag* is introduced. While this is apparent from the negative sign on the x term of the Lorentz transform, recall that it also depends on velocity sign conventions and the reverse transform therefore switches sign, so when students are learning many new things it is easy to become confused. If the reader prefers to be particular about grammatical symmetry, a phrase such as *leading clock retardation* could be used, but it doesn't seem to the author as important as something simple and memorable.

One of the most interesting applications of "leading clocks lag" is a simple derivation of de Broglie wavelength. De Broglie's original derivation [4] was based on the Lorentz transform, and our present version essentially follows this but with the use of our memory aid to simplify the derivation. The derivation is sufficiently obscure in history that it was recently recounted in a modern paper [5], and is completely omitted from many "explanations" of the "derivation" of de Broglie wavelength [6] [7]. This derivation also answers questions that have appeared on various physics student blogs asking how to perform the Lorentz transformation of de Broglie wavelength, which as it turns out is not the clearest way to ask the question [8].

2. Types of clock measurements

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Two on one clock measurements

We must use two or more clocks in an Einstein synchronized reference frame to observe time dilation in a single moving clock under observation. Observations must be made only by reference frame clocks which are local (i.e. adjacent) to the clock under observation (i.e. as they pass). No remote observations are allowed. I call this *2-on-1* clock measurement. In Figure 1, A with two clocks makes a *2-on-1* measurement of B's clock.

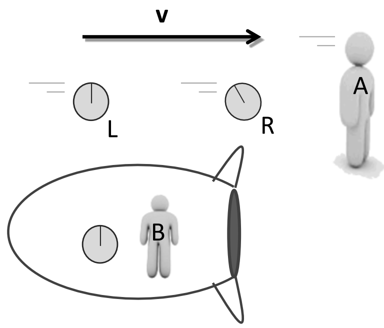

Fig. 1 – Clock measurement, *2-on-1* A observes B, or *1-on-2* B observes A, from point of view of B

In the figure, A has two synchronized clocks R and L. They don't appear synchronized to B because of relative motion (the diagram is from B's point of view). As B passes clock R, A records both clocks and labels the readings R and B_R. Similarly as B passes L, A records L and B_L. Since A is using an Einstein synchronized frame, we can infer using $\gamma = 1/\sqrt{(1-v^2/c^2)}$ that:

$$\begin{aligned} &\Delta T_B = \Delta T_A / \gamma \\ &\Leftrightarrow (L-R)/(B_L - B_R) = \gamma \end{aligned} \qquad (1)$$

One on two clock measurements

It is well known that the observer B, if using two synchronized clocks, would reach the reciprocal conclusion, that time is dilated in A's frame. But suppose B measures with just the one B clock shown, against the two clocks R and L in passing? This measurement does *not* use an Einstein synchronized reference frame. The exact same times will be recorded as by A in the previous example, and B would conclude that time runs faster in A's frame. We may understand this as being similar to a traveler in an airplane crossing time zones. In B's frame, L and R are not synchronized. The leading clock, the one B encounters first, R, is lagging by some amount. As B passes clock L, the time difference includes not only the general rate of time passage in A's frame, but also adds the lag time that B sees between the two clocks R and L.

This *1-on-2* sort of observation is useful for determining what time it would be if B suddenly dropped into A's frame. It's used every day by airline passengers and schedulers, and could be used by the traveling twin in the traveling twins puzzle to determine the age of a stay-at-home twin. But it does not indicate the rate of ticking B sees in A's clocks when making a well-defined measurement using a reference frame with synchronized clocks.

3. Application to de Broglie Waves

De Broglie waves are based on Planck's idea of quantization of energy for photons, but applying the idea to matter by assuming matter has an associated "wave." De Broglie takes Planck's relations as given (we use f to distinguish frequency from velocity):

$$\begin{aligned} E &= hf \\ \lambda &= h/mv \end{aligned} \qquad (6)$$

The energy E is taken to be the relativistic energy $E=\gamma mc^2$. Wavelength is Planck's constant divided by momentum. For light, the speed c provided an obvious physical relationship between frequency and wavelength, but for matter the frequency is non-zero even when momentum is zero, and wavelength is infinite under those conditions. If velocity increases, frequency changes only slightly as it is dominated by the large rest mass energy, but wavelength rapidly decreases. This is not intuitive and bears to relation to light wavelengths, and is a source of confusion when performing coordinate transformations, because the de Broglie wavelength obviously cannot be handled as a real physical length.

Figure 2 shows de Broglie waves schematically in the rest frame of a particle. We should qualify this figure in several ways. Quantum particles do not strictly speaking have a known certain rest frame due to the uncertainty principle, but taking such complexity into consideration does not really add anything to the derivation of de Broglie wavelength. To the extent we approximately know the rest frame of a particle the de Broglie wavelength is approximately infinite. That is, the particle's associated wave vibrates, or you might think of it as "flashes," simultaneously throughout all space. A peak of the wave (dashed line) is like the tick of synchronized clocks. For illustration in the figure clocks at various points in space on a peak in time are shown as synchronized.

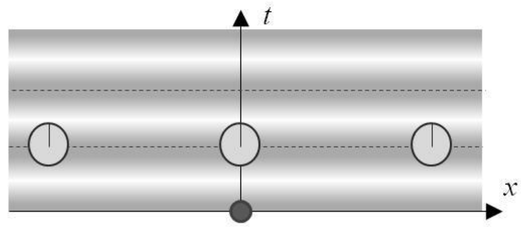

Fig. 2 – de Broglie waves in the rest frame of a particle

Note that de Broglie waves are not specifically shaped as probabilities like Schrodinger waves. De Broglie simply visualized a wave associated with a particle, and the probability interpretation came later. Also, de Broglie waves are based on the total energy of the particle including rest mass, while Schrodinger waves are based on kinetic energy. Since energy differences are all that matter physically, the two can be seen as differing only by a choice of measurement coordinates. It would appear that the frequencies are different, but again it is only differences that matter, and wavelengths, which are the property associated with physical manifestations like interference, are the same for de Broglie and Schrodinger waves (phase velocities, a calculated quantity not directly measurable, are different).

Notice that the de Broglie rest frame constitutes an Einstein synchronized reference frame. If we observe from another inertial frame using a point observer as shown in Figure 3 (we don't have any way to detect the wave peaks at other than a point using quantum measurement techniques), the wave peak clocks cannot all be synchronized, and thus the de Broglie wave cannot "flash" everywhere at once. As leading clocks will lag, the wave peak will occur a little later at leading points as viewed from a moving frame. Furthermore, since this is a *1-on-2* measurement process (or *1-on-many*), the de Broglie frequency appears higher by a factor of γ rather than lower.

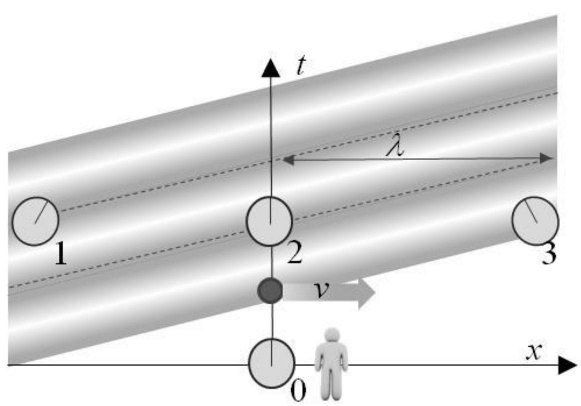

Fig. 3 – de Broglie waves seen from another inertial frame
(Wave peak clocks 1 and 3 do not appear synchronized with observer's clock 0. The vertical time axis has higher values at the top, and therefore the leading clock 3 is lagging, and corresponds to an earlier wave time.)

If the de Broglie wave has frequency f, it will have period $1/f$, or in the observer's frame $1/\gamma f$. If we set the lagging time to the period and the wavelength λ to ΔL and solve for λ, we will arrive at the de Broglie wavelength:

$$\begin{aligned} &1/\gamma f = \gamma v \Delta L / c^2 = \gamma v \lambda / c^2 \\ &\Leftrightarrow \lambda = c^2 / vf \end{aligned} \tag{2}$$

Using $E=hf$ which alternately we can write as $f=E/h$ or $f=\gamma mc^2/h$, we can obtain the more customary form:

$$\begin{aligned} &\lambda = c^2 / v(\gamma mc^2 / h) \\ &\Leftrightarrow \lambda = h / \gamma mv \end{aligned} \tag{3}$$

As to the Lorentz transformation of λ, it is clear from the derivation that it does not have two physical endpoints in the rest frame of the particle but is already the product of a Lorentz transformation of simultaneity, and must be re-computed rather than illogically double transformed in order to get to yet another reference frame.

4. Conclusion

A new memory aid "leading clocks lag" puts the intuitive visualization of relativistic time skew on an equal footing with time dilation and length contraction. By understanding the nature of Einstein *2-on-1* clock measurements vs. the *1-on-multiple* clock sampling of de Broglie waves, we reconstruct the derivation of de Broglie wavelength as an artifact of the Lorentz transform, not a length to be further transformed, and understand the increase rather than decrease of de Broglie frequency with velocity as a consequence of non-Einstein measurement rather than some sort of puzzle or exception. The author invites readers to try this explanation in their class or physics club and either publish student response in this journal, or contact the author to consolidate results.

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