Non-Linear Market Theory

Robert Shuler - September 2003

Non-linear Market Theory is a term I use for a collection of ideas, all of which defy linear arithmetical logic. That is, the returns, or the value, of two investments will not add together. They will be worth more inside a large portfolio than they are individually. During high volatility, this difference is larger.

Therefore, it is reasonable to expect novice investors will lose money on their first few investments when their portfolios are small. Predictions of the theory agree with some of the most basic yet puzzling financial findings. It also predicts that very long-growth returns emerge through pooling, and have a worth so high that, like a sort of financial singularity, they can distort the operation of the economy and society.

The first section discusses the statistical contribution to non-linearity, and the second section the value of growth, both involving some simple math. A third section discusses social implications, and a fourth enumerates conjectures and puzzles remaining. Finally, there is some portfolio advice and a conclusion.

MEDIAN RETURNS

Computing Future Returns. To compute our expected returns (or profits) from investing taking into account the uncertainty of whether investments will go up or down in value, we look at what the probability of various returns has typically been. For example, a no-risk investment might return 4% every year with a probability of 100%. That is a very simple probability distribution which has a so-called "expected" or average return of 100% x 4% = 4%.

Now suppose we have invested in a mutual fund, which goes up in value about 20% in a good year, and down in value 10% in a bad year. For simplicity, assume half the years are good and half bad, so that each return has a probability of 50% or 0.5. This is a "binomial" distribution, because it has two possible outcomes.

Say you are betting on coin flips with a generous friend who pays you $20 when you win and collects only $10 when you lose. Your expected return is 0.5 x $20 + 0.5 x (-$10) = $10 - $5 = $5 for each flip. If you flip twice the probability of winning both times is 0.5 x 0.5 = 0.25, and the amount you win is $20 + $20 = $40. The following shows a complete table of all possible results after two flips:

Probability	Return
0.25	$40
0.50	$10
0.25	-$20
Total probability = 1.0 (100%)	*Average = .25(40) + .5(10) + .25(-20) = $10*

Table 1. Arithmetical Returns from Flipping Coins, 2 Trials, Payout $20 Win & $10 Loss

This table also is a "binomial distribution," and statistics books are filled with more extensive tables for more flips. The defining features of a binomial are that the probability at each flip is independent, i.e. does not have anything to do with previous flips, and the return at each flip is also independent. You can find these conditions in any textbook that discusses distributions (e.g. Baird).

The binomial is a discrete distribution. If you flip the coin infinitely many times so that you can record a continuous probability distribution, it will be shaped like the classic “bell curve” you have heard of, and is called a Gaussian, or “normal” or “standard” distribution. So, the Gaussian is based on the idea of the binomial. Gaussians apply to many natural phenomena, have special properties like finite variance, and have well behaved and well understood analytical properties.

One important property of such a standard distribution is that the average value is also equal to the middle value, or median. The median is the value such that the probabilities of getting a return above or below it are the same. In the above example, the median return is $10 (after two flips), same as the average. The average is also called the “mean” or “expected” return.

The “mode” of a distribution is the most frequently occurring or most likely value. It is easy to see the mode of the above example is also $10, the same as the average and median, and this is true of all Gaussian (normal or standard) distributions.

Distributions for Growth Investments. The conditions for applicability of binomials or Gaussians, however, admit only independent arithmetical returns, not growth or geometric returns in which the value of each return depends on all preceding.

Economists focus on the "percentage" of the return, which "appears" to be independent. That seems to suggest the average return (mean or expected) can be used with the standard deviation to fully characterize the probability distribution, and that the average return from a combination of two investments will be the sum of the individual average return from each.

A very simple numerical example will illustrate. Assume you have an investment which 50% of the time returns 10% and 50% of the time returns -10%. Not a very good investment, the "average" return is zero, but it is easy to calculate with these numbers in your head.

Starting with $1 invested, at the end of one return period (e.g. year) you have either $1.10 or $.90, with a 50% probability of each. This is the returns "distribution."

At the end of two periods, you have a 25% chance of being at $1.21, a 25% chance of being at $.81, and a 50% chance of being at $.99. An up year followed either by a down year, or vice versa, produces the $.99 return. Here is the distribution in table form, with the returns expressed as percentage change:

Probability after two periods	Return as relative percent	Total Portfolio Value
0.25	21%	$1.21
0.50	-1%	$0.99
0.25	-19%	$0.81

Table 2. Growth Returns, 2 Periods, 10% Win or Loss

As will be obvious, +10% followed by -10% does not add to zero. Neither does –10% followed by +10%. However, if one computes the average return, it really is $1. This is computed by summing every possible outcome, weighted by its probability of occurrence, as .25*$1.21 + .5*.99 + .25*.81 = 1.

What has happened is that the very high returns achieved by a lucky few investors who experience serial wins are offset by a typical middle-of-the-pack return that is a slight loss. The

"median" return is no longer equal to the mean (average), as it would be for a normal arithmetical binomial (or Gaussian). It is 1% less.

Now consider a pretty typical hot growth stock which is up 60% in a good year and down 40% in a bad year, giving a respectable average return of 10% per year. Here is its two year expected returns.

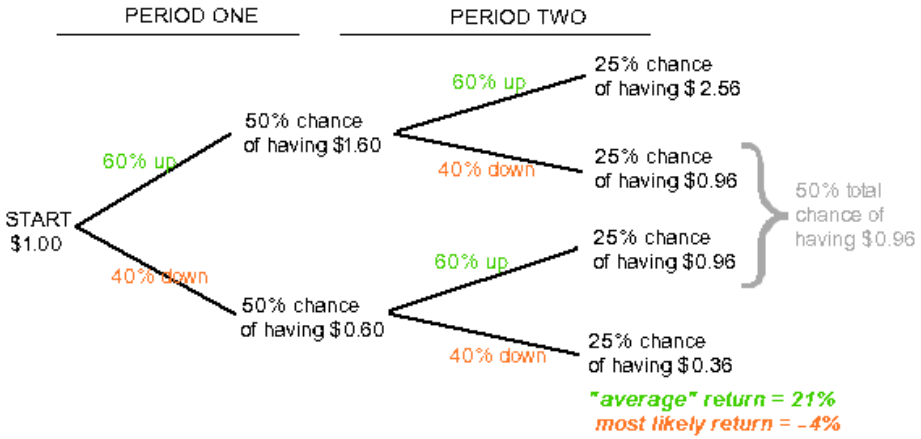

Figure 1. Growth Returns from Investment with +60% -40% Payout (average 10%), 2 Periods

It's easy to see that if there are a bunch of stocks like this one, and you and three friends each pick a different one and invest $100,000 in it, at the end of 2 years one of you has over a quarter million dollars. The others all lose money. One loses $64,000. The winner has made so much that the "average" return for the four of you is still a gain of $21,000.

Would you take this bet? I wouldn't. But in the remainder of this discussion I will show you how to take investments like this and capture very nearly the predicted average gain. And I will show you some bizarre effects the "hidden losses" and "unbalanced gains" have on markets and society.

The Lopsided Bell Curve. Economists go to some amazing lengths to keep up the fiction of average returns. Not only do they ignore the conditions for which Gaussians are valid and compute with percentages, but they also use logarithmic plotting, because a growth return plotted normally produces a skewed shape that does not look at all like a bell curve, as shown in the chart below.

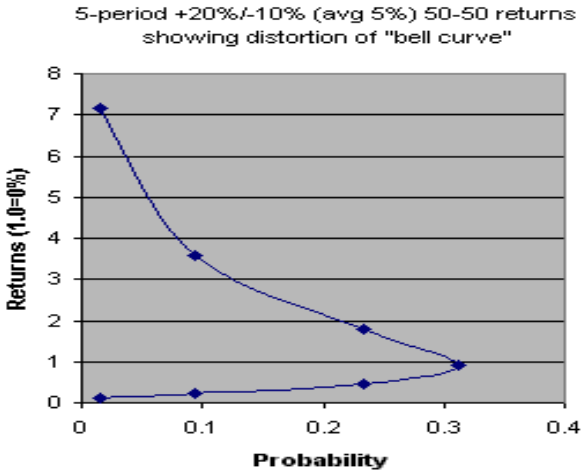

Figure 2. Growth Binomial Distrubution Showing Distorted Bell Curve

You will see right away that this type of investing, akin to "letting your bet ride" on the roulette wheel, makes a few people rich just by random accident. If you carry the distribution out to larger numbers of periods, it appears to eventually bankrupt everyone. However, the rich do not let their bets ride. This brings us to consideration of portfolios and volatility.

Volatility. Non-Gaussian distributions are much more analytically difficult, which is probably why they have been slow to catch on, even though certain ones like Pareto-Levy have been known for years to be more reflective of actual market returns.

I spent several weeks writing software to help me visualize the mean vs. the median in various situations. The first thing I learned is that for low volatility, the average and median are very close. At high volatility, i.e. large swings up and down, the median is very much depressed. It can go negative even for an investment with a respectable average return, as in the 60-40 example. There is a range within which one can partly offset the drop by keeping part of one's portfolio in a risk-free investment and re-balancing at each period (e.g. annually). However, eventually even this does not work. The chart below shows an example of median vs. average returns at various volatilities, with and without re-balancing.

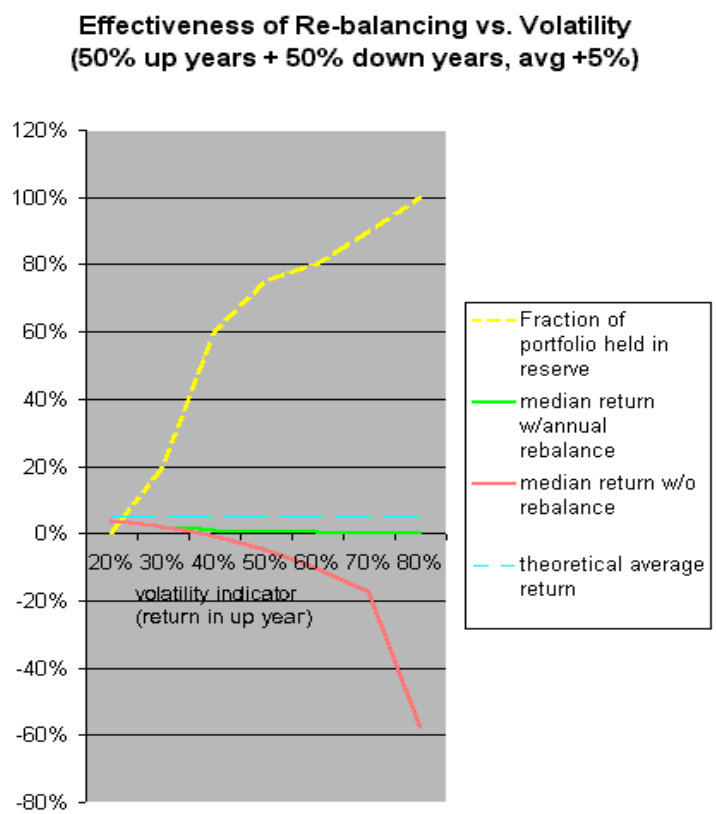

Figure 3. Effectiveness of Re-Balancing vs. Volatility

Portfolios. Next, I wondered what was the effect of holding more than one similar but uncorrelated investment in a portfolio. Holding correlated investments is obviously equivalent to just holding more of one of them. With the caveat that it is known that uncorrelated investments can become correlated in extreme market conditions, I calculated a wide range of cases.

Holding two items confers marginal improvement. Three items confer noticeable improvement. By the time one is up to 20 uncorrelated investments, one is getting very near the average return. Moreover, at 40 items one gets the average return even under extreme volatility.

The way to visualize this is that a large portfolio is more likely to "capture" the small number of investments that produce the stellar serial wins, and pull up the average.

Equity as an Option. Despite the bleak picture I have painted, none of the returns take the portfolio value below zero. This is because as I learned from a survey paper by Zingales, equity is actually an "option" whose strike price is the corporate debt. This is what was codified into law in 1850 as the LLC (Limited Liability Company). If I make an investment without incorporating and borrow money and lose it, or someone sues me, then certainly I can lose more than my original investment. But not so a corporation. This has an enormous effect when

it comes to the Crash Rate Analysis (a separate topic), but in our current context it has the more modest, but substantial, effect of limiting catastrophic losses.

Summary of Median Returns. So we have a situation in which some investors will get rich by luck, and if they then manage their portfolios properly they will be able to keep their wealth. Most people will experience small losses, or less than average returns, and simply get discouraged and figure that it must be an inside game. A few people will experience spectacular losses. No one will lose more money than they put up directly (unless they buy on margin, which appears foolish given these odds), but society may have to bear the excess costs of extreme corporate misbehavior due to the limitations on liability.

Some of these problems are surprising, but none seem insurmountable. In the next discussion we may not be so lucky. That finishes our statistical discussion. In the next section I will assume that investors are smart enough to pick an average or median or other "typical" return appropriate to their portfolio composition to use in determining the value of investments, and I will treat returns as a constant percentage year after year.

THE VALUE OF GROWTH

What is the present value of an investment in a company whose earnings will grow by some amount in the future? What should I pay for future earnings? How much would I deposit to earn that much interest?

Assume some interest rate R, also called a discount rate, which can be used to find the "present" or "discounted" value of future earnings. If you expect to receive some future earnings E_N in a period N years hence, then the NPV (net present value) is given by the following expression:

$$NPV = E_N/(1+R)^N$$

In other words, NPV is the amount of money you'd have to place on deposit today in order to withdraw E_N dollars N years hence.

The question we want to consider is, if we have some initial earnings stream E_0, which is growing at rate G, and the growth continues for N years, what is it worth?

Sometimes economists proceed by making structural assumptions about how the growth is produced through internal re-investment in proprietary opportunities. I don't care how the growth is obtained, and wish only to make a "black box" analysis, so I will stick with the parameter G. The definition of G is simply the annual growth in earnings such that $E_{i+1}=E_i(1+G)$. This implies that $E_N = E_0(1+G)^N$, and so we can write an expression for the NPV of each future earnings amount:

$$NPV_N = E_0 \{ (1+G)/(1+R) \}^N$$

The value of an investment which produces a growth stream of earnings $E_i = E_{i-1}(1+G)$ is then the summation of NPV_i for i=0 to infinity.

In order to evaluate the summation, I use what I call a "liquidation value model." This simply assumes that all earnings are re-invested up until the point at which growth ceases, and at that point the investment is liquidated for the interest rate determined value of its then-present

earnings E_N/R. It remains only to calculate the NPV corresponding to E_N/R, which is easily seen to be:

$NPV(E_N/R) = (E_0/R) \{ (1+G)/(1+R) \}^N$

The Growth Equation. To simplify, multiply inside the brackets by (1-R)/(1-R) and discard the relatively unimportant second order terms (R^2 and GR). This gives the expression:

$NPV(\text{growth investment}) = (E_0/R) (1 + G - R)^N$

It's possible to generalize this to encompass dividends which are re-invested. I'll spare you the somewhat complex algebra and just present the highly intuitive result. If Y is the dividend yield, then:

$NPV(E_0,R,G,Y,N) = (E_0/R) (1 + G + Y - R)^N$

Dividends. So obviously G+Y is the effective growth rate. A fact less well known than you might expect, which investors have ignored at their peril in the last few years. Henceforth I will just use the term G which in the case of a dividend paying company you can take to mean G+Y.

PE Ratio. If one wishes to look at PE, one just divides the entire expression by E_0. In classic value investing theory, the base PE ratio is 1/R, with higher PE's being a premium attributable to growth. In my formulation, it is obvious that the PE premium (ratio of growth PE to non-growth PE) is exactly $(1+G-R)^N$ where G is the expected (average or median, whichever you think you will achieve) growth rate, R the expected interest rate, and N the expected number of years excess growth will continue.

A spreadsheet which fits this equation to about 30 equities was developed, and the results indicate that in most cases investors seem to expect growth forecasts to hold for about 5 to 10 years. In a few cases, notably Dow component Hewlett Packard, the model suggested investors have priced the stock for actual decline, instead of the forecasted growth. Indeed, a week after making this finding, HP announced yet another disappointment in earnings, and the price declined further.

SOCIAL IMPLICATIONS

Growth vs. Corporate Governance. Given the earlier discussion of median vs. average returns, and the portfolio effect, we conclude that managers of large portfolios should be using a higher value for G and investors with a small number of holdings, or predominantly one holding, should be using a lower value for G such as the median. This accounts for two observations. First, studies have shown that the average investor-managed brokerage account loses money, and has to be replenished from external funds. So mutual funds do better than individual investors. Second, index funds do better than managed mutual funds, by about 1% or so.

With the growth equation, we see that the sometimes small delta between portfolios of one size or another is raised to the N^{th} power, so it makes a big difference.

The downside is that corporations are supposed to be little democracies, governed by their shareholders. Shareholders have other interests in life, and until recently it was not uncommon for an environmental or ethical cause de jure to hijack an annual shareholder meeting, and have an impact upon corporate behavior.

However, in 1986 only 30% of public company equity was owned by institutions, whereas today that figure is 60%. It has risen due to the favorable returns afforded to large portfolios by the basic mathematics of probability, and its amplification in the growth equation.

It is practically impossible for an investor with 40 equities (about my portfolio size) to keep up with and vote intelligently in the affairs of 40 companies. Mutual funds, under current custom and law, do not allow shareholders to vote corporate shares, and do even make public how the shares were voted!

Does this have an effect? Of course it does. Take the recent HP-Compaq merger, which produced the current incarnation of the money-losing HP. It was opposed by both founding families of HP. And it was opposed by many small shareholders, including myself. I had holdings in both companies. I didn't want them to merge. I wanted them to stay focused. I knew positively the merger would cause near term efficiency loss, market disarray, and loss of market leadership to Dell. Which was exactly what happened. I had been betting on HP's printer business, and Compaq's strong local investors who had always before pulled the company out of trouble. In the merger the printer business was diluted, and Compaq's local investors were rendered moot. I guess they gave up and bailed out.

Agency. Economists have done a pretty good job of studying corporate governance. They refer to this as the agency problem, the problem of how to keep the professional managers you've hired from running away with all the profits and not managing the company in the investors' interests.

What they have not done is study the double-agency problem of investing in mutual funds. The fund manager is a second agent, who also does not have the investor's interests completely at heart. Even worse, the fund manager lives is a skewed world. He or she is disproportionately affected by corporate profits in general, and by relationships with corporate executives, than are shareholders. An ordinary investor generally has some other income, like a job, and a healthy interest in ethics and the environment. A fund manager has a disproportionate interest in corporate profits. Moreover, the fund manager makes his living out of the difference between the returns available to individual investors with small portfolios, and the returns available within large portfolios. The fund manager, according to game theory, will take every opportunity to improve his own position at the expense of the investors. This includes monopolizing their voting rights.

Longevity of Growth. It is in regard to the longevity of growth, the size of N, that things really get out of hand. All the aforementioned problems are human problems with human dimensions, capable of being studied and managed by humans. The longevity of growth is not. It has infinite, or supernatural dimensions.

Corporations may outlive individuals, but in practice only a few live more than a century. Those that have, like GE, have more in common with diversified mutual funds than unified business enterprises. The real gain in lifespan comes through large institutional portfolios. The largest gains come through index funds. And the longest lived index funds will exist with stable, large countries which devote a substantial fraction of their resources to military and other means of exerting and maintaining their market dominance.

In other words, if we examine the parameters G and N for a portfolio, we find they are both larger. G is larger because of the median calculus we've outlined above. N is larger because

the portfolio may outlive its components, just as the Dow Jones 30 Industrials and the S&P 500 have outlived most of their components.

Economists routinely calculate average growth rates for data going back into the 1800's. Shiller's data for the S&P going back to 1871 is available on the web, and I have used it to study the equity premium. Below is a plot of G (earnings growth, highlighted with boxes) from Shiller's data.

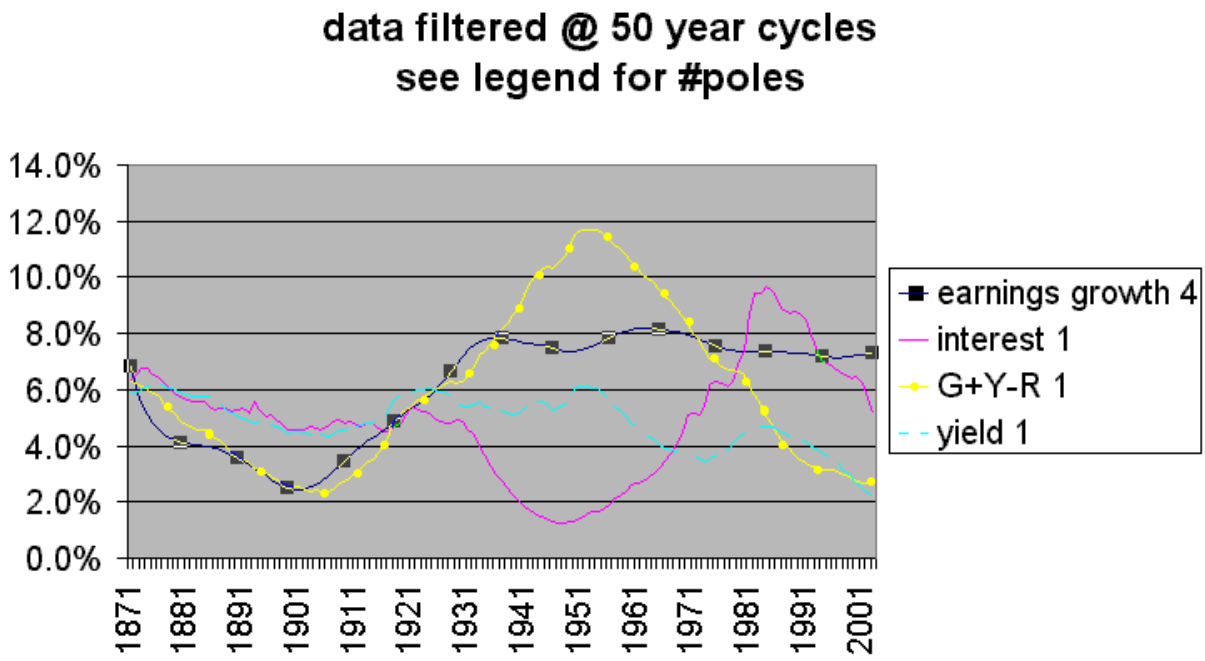

Figure 4. S&P 500 Earnings Growth, Yield, Interest & Equity Premium, 1871-2003

These curves are filtered with simple low pass filters, not with the "moving average" and similar filters ordinarily used by technicians. Low pass filters make long term trends much more obvious. Notice that not only has growth remained positive during this entire 132 year period, but the term G+Y-R (yellow line highlighted with boxes) has remained significantly positive, implying a credible basis for believing N >> 132 for the S&P 500 index.

The Equity Premium. The yellow line G+Y-R is very nearly the much-studied "equity premium" first documented by Mehra and Prescott. The only additional component of the equity premium is the relative change in PE, which has oscillated between 12 and 25 for most of this period, rising recently. The recent rise was offset by a drop in yield, so that there was no net effect on the equity premium.

The curve does not provide a rationale for the equity premium, it merely allows us to plot and examine it. As noted by Mehra & Prescott, it was smaller before 1930 than it has been since. In the 1930's, the federal government began to actively regulate interest rates, instituted FDIC protection to make a true risk-free return possible for the first time (during the 1920's losses from bank failures amounted to 2% per year, increasing to 3% during the depression), and also regulated the degree to which banking funds could be invested in equities.

The Law of Equal Returns. The reason the equity premium is such a puzzle is because of an assumption that permeates economic theory called the Law of Equal Returns. It basically

postulates that if an investment is available with a higher return G than the normal R, investors will bid up the price of it until the effective return, allowing for an appropriate risk premium, is reduced to R. The observed equity premium is an order of magnitude higher than what is justified for a risk premium, and thus the "puzzle."

Practical Infinity. If we assume the total S&P will continue to offer a premium of about 6% for the next 132 years, then the (1+G-R)**N term which provides a multiplier to figure the premium "growth" PE needed to annihilate those returns (over the ordinary 1/R "value" PE), we find that the PE multiplier needed is 2189. In other words, the stocks in the S&P 500, under perfectly reasonable assumptions about the future of our country, are worth in the form of the index at least if not individually, over 2000 times what they are currently selling for. The S&P index is hovering a little below 1000 at the moment. This implies that under the assumption of efficient markets, if investors were fully informed of the correct way to value growth, they would on the next trading day instantly bid up this index to around ***2 million!***

In all probability, this exceeds world GNP. In other words, it is a number such that when compared to any other real present financial value in the whole world, including the sum total of the world's economic output, it is found to be "much greater." In *otherworldly* words, it for all practical purposes has the value "*infinity*."

There are three things here that it took me some weeks of contemplation to absorb and realize, so I will take the liberty of pointing them out, even if perhaps they might be obvious to you.

1. The first is that in the practical matter of pricing stocks, infinite funds are not available to purchase them, so what the growth equation applied to long-lived portfolios really says is that the value is "indeterminate." It is arbitrary, set by supply and demand, and could be anything.

2. The second and more shocking is that the force of the infinite value is really there anyway, even if investors do not currently possess the funds to bid it up that high. It makes itself felt over time. As the returns accumulate, the investments with premium returns suck all of the capital out of every other type of economic activity. This is the effect of the law of equal returns. The presence of a "long growth" return is like a black hole. No matter how high the price is bid up, the returns after some period of time still amount to a premium. It doesn't matter whether this return is ethical or socially valuable or aesthetic. The flaw is in mapping everything to a single scale of monetary value. That value eventually nudged humanity with infinite force to create laws to favor corporations (LLC and corporate personhood) because these laws extend the value of N and increase the value toward infinity. Further laws and customs favored the collection of corporate equity into giant, ungovernable funds for the same reason. It is my conviction that the entire resources of the U.S. political process, both parties, and it's significant military, are now entirely at the disposal of the economic drive to increase N, and that moral and ethical justifications have been invented to serve and rationalize this process.

3. Finally, the third and most shocking realization is that the very idea of an infinite economic return crosses over from the human realm to the divine. It is a promise of utopia. It is a challenge to God, whose province alone it is to contemplate infinity. Even the ancient pagans were smart enough to know that challenges to the godhood would not go unanswered. But modern mainline Methodists and Baptists and Presbyterians, even and especially of the fundamentalist and or evangelical variety, have totally allied themselves with the forces of this economic "great attractor." The "sop" of a few issues they care about, together with the "great promise" of a better world through the economic efforts of man, have led them "firmly astray."

Revised Law of Returns. In light of what we now know about growth returns, the so-called Law of Equal Returns must be revised. This principle didn't anticipate that by the devices of corporate personhood, limited liability, and pooling equities that returns could be made nearly infinite, and therefore un-equalizable. If investors cannot bid up the value of an investment with a higher return enough to equalize the return, then the economy never returns to a balance in which all investment opportunities attract capital in proportion to their worth. Instead, just a few investments with the highest returns will attract all available capital, and other investments will be abandoned.

In our re-statement, there is no realistic expectation of equalization of returns. We simply have the situation that all investment capital flows to the highest return. This inflow of investment capital can even empower the continuation of the differentially higher returns.

The law of highest returns has two effects. First, within each type of investment, one will naturally prevail. The one that offers the highest returns. It need not have high quality, treat workers or customers well, or have any other property than just the highest returns. Second, among investments of different types, the lower returning ones will experience extreme neglect until under investment reaches crisis proportions, creating at least temporarily a sufficiently high return to attract capital.

This is a rather bizarre situation. If even a modest pool of investment opportunities is found which has higher long-growth returns than the norm in the economy, it can cause all other parts of the economy to become extremely distorted.

The Life of Companies. Companies can of course fail. Xerox, for example, despite making investments in technology that led to the wide adoption of desktop computers, gave the technology away to Apple, who also failed to defend it aggressively so that it was taken over and copied by Microsoft.

However, Microsoft has successfully defended its hegemony even as the technology of interest has changed through its massive ability to invest for long periods before requiring profitability, even to the point of giving away free product. GE, already noted as the longest running Dow 30 component, also successfully negotiated investment in one new technology after another, moving from electric power to aircraft engines, plastics, nuclear power, and financial services.

Wal-Mart has been so successful at delivering higher returns for so long that it now threatens to become the sole supplier of consumer goods and even groceries in many towns, illustrating our point about the highest return rate within a sector. Competitors like Target are now forced to match Wal-Mart move for move, even into the grocery business, or face the evaporation of their investment capital.

Returns Substitution. Just as a company facing its demise may engage in asset substitution in an attempt to transfer its creditors resources to its own balance sheet, a company facing lower than the best returns available may practice *returns substitution* by purchasing equity in investments with higher returns in lieu of prosecuting its own business, thus transferring its competitors' resources to its balance sheet. The most famous and successful example of this is Berkshire-Hathaway, which long ago ceased operations within the textile industry and became a sort of holding company. Major companies like Intel and Microsoft have significant venture capital operations. Although they have not abandoned their core business, they expect one day superior returns will emerge among the startups they sponsor.

Exaggeration of Returns. When corporations increase their market holdings, and especially if they leverage them, positive feedback can occur in which good market returns begat more good market returns, and vice versa. When fraudulent or inaccurate returns are included in the mix, such as from Worldcom or Enron, the total amount of the earnings misstatement may be exaggerated by the market due to this positive feedback, as well as the general tendency to try and emulate the companies with highest returns. Analyzing the problem of exaggerated returns was actually what led to the development of non-linear market theory.

Unethical Returns. Even more shocking is the realization that there is no effective way to combat unethical high returns if they are long-growth returns. There is no penalty high enough to overcome an "infinite attractor." Probably this is why the "War on Drugs" has never made much progress. The attempt to make the penalty "nearly infinite" has caused the U.S. to incarcerate a higher percentage of its population than any other civilized nation. A more fruitful approach might be to find a way to reduce the returns. However, reducing the returns of drug trafficking is objected to on moral grounds, as it usually involves some sort of semi-legalization.

Once established, whether legal or illegal, long-growth high returns are very difficult to oppose. Even if one wins a temporary battle, if the returns are not effectively neutralized, then capital continues to accumulate and reinforce the social and political climate that permits the returns. The subject industry is said to provide jobs and other secondary benefits. It certainly contributes heavily to its political allies. Large numbers of people will have naturally come to own its equity shares. There will be impacts to pension funds that happen to have invested in that particular equity. It will be very hard to completely remove the high returns from an industry like tobacco, or computer software.

The Evolution of Capitalism. There are technologies on the horizon that might be capable of producing astronomical long-growth returns: genetic engineering for example, or nanotechnology, or even robotics if the technology ever matures. The emergence of a new group of very high long-growth returns, far from being a utopia, might destroy investment in mundane services like electric power, municipal utilities and environmental preservation. It might even make it hard to get many types of consumer goods that we now take for granted.

In short, because of the law of "highest" rather than equal returns, the end-stage of evolution of capitalism might be as depleted of choices and basic services as the end-stage of socialism. Abundance might be more a product of transitions, not of equilibrium.

Are we doomed by the law of highest returns? Probably not. If one looks at biological systems, which have been subject to similar laws of natural selection for millions of years, there are stable niches with enormous diversity. But these take a long time to evolve. For thousands of years following major upheavals or migrations, diversity can be greatly depressed. Over very long times, there seems to be some drawback to the superiority of long-growth returns. It might be possible to explain this with the Crash Rate Theory alluded to earlier.

Summary of the Value of Growth & Social Implications. The premium multiple for the value of a growth stock is proportional to the differential returns it offers (1 + G+Y - R) raised to the power N where N is the number of years growth will continue. The value of such investments adds non-linearly in portfolios because of the increased expected N of the entire portfolio, creating an incentive to pool resources, which makes them ungovernable. Finally, the resources themselves "come to life" in the form of the LLC and corporate personhood legal doctrines, and ultimately begin to influence the political process so as to further increase N.

CONJECTURES & PUZZLES

The Risk Free Rate & the Equity Premium. Despite my re-casting of the law of equal returns as a new "law of highest returns," non-linear markets can make capital available for lower return investments by several methods.

First, as noted earlier, in higher volatility ranges it is to investors' advantage to allocate a substantial fraction of capital to a risk free investment to be used in periodic re-balancing. Investors may even tolerate slight negative returns on this capital if it is truly risk free. Based on the amount of this capital investors make available (demand) and the supply of low risk investments, the market would determine the risk free rate of return.

Second, and subtler, is the capital made available because of the heterogeneity of returns. If all investors do not hold a single index fund (and they don't, for whatever reason, possibly because this would destroy market efficiency and corporate governance) then at any one time as investors evaluate their performance over the last 5 or 10 years, 50% of them would find they are getting median returns or less. We have already established that the median will be less than the average returns. Even in an up market, some investors would be making less than the risk free return, and some investors would find they had been losing money over 5 to 10 year periods. A substantial fraction of capital, 10%, 20% or more could be coming "off the table" each year for this reason even assuming Gaussian distributions. With Pareto and other distributions more reflective of actual market returns, the percentage is larger.

Third, the government can and does intervene to make available funds (from tax revenues) for short term lending at a rate the government decides, namely the Fed's overnight discount rate. Through this and other methods, the government forces capital to be available for lower return investments.

The Growth-Yield Conjecture. Bond returns appear to be inversely correlated with growth, both with respect to individual companies, and the economy as a whole. This can perhaps be explained in terms of individual companies by arguing that if a company has good growth prospects, investors may feel they are more likely to be repaid.

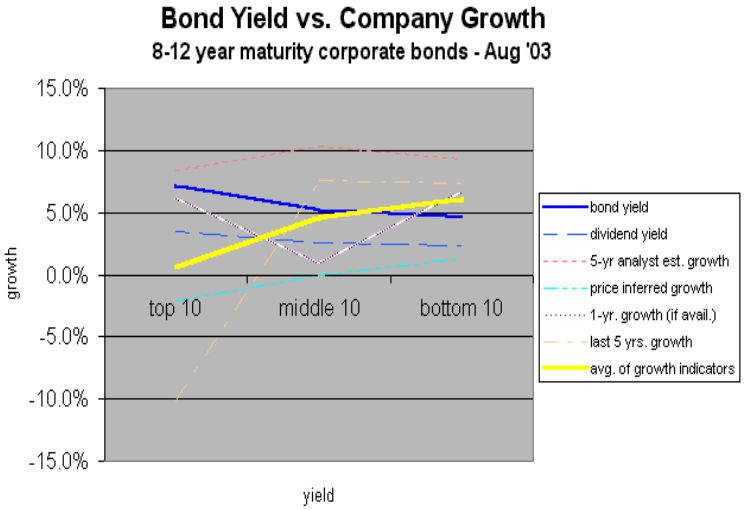

Figure 5. Growth vs. Yield for 30 Companies

The chart at above plots bond yield vs. growth for 30 companies. Since growth is somewhat ambiguous, several measures are plotted, and then an average of these is taken. While some

measures of growth vary slightly from the conjecture, none varies completely, and the aggregate agrees well.

The chart below is a plot of excess growth (G-R) vs. real interest (R-I where I is the inflation rate) for the 132 years of the Shiller data, filtered to remove short-term variations. The inverse correlation is striking over the entire period.

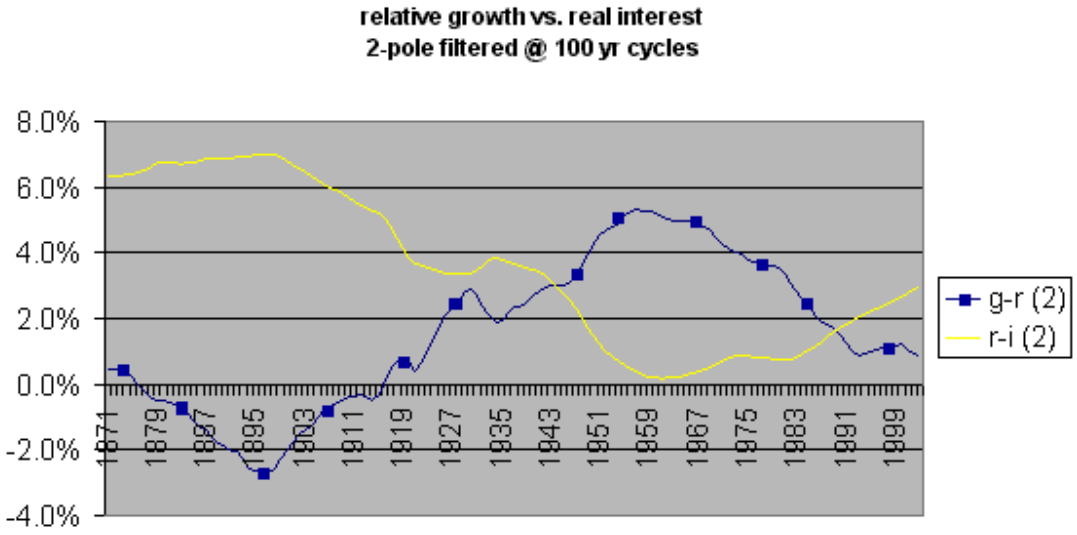

Figure 6. Excess Growth (G-R) vs. Real Interest (R-I) for S&P 500, 1871-2003

From the point of view that low real interest stimulates growth, the chart makes perfect sense. But from the point of view that capital should be attracted to the higher return until the lower return is driven up to compete, the chart is inexplicable. We have seen that capital cannot "bid down" the high rate of return by bidding up the price to the extent previously thought. Is there some as yet unknown principle by which it cannot really raise a low rate of return by bidding down the price either? During most of the period in which G-R is positive, there was active government meddling in interest rates, which might provide an explanation.

Butterfly Distributions. Long period (a year or more) returns appear to follow what I call a butterfly distribution. Rather than a peak near the middle or average return, there is a dip there. Consider the following histograms of 1, 2 and 4-year returns from the Shiller data.

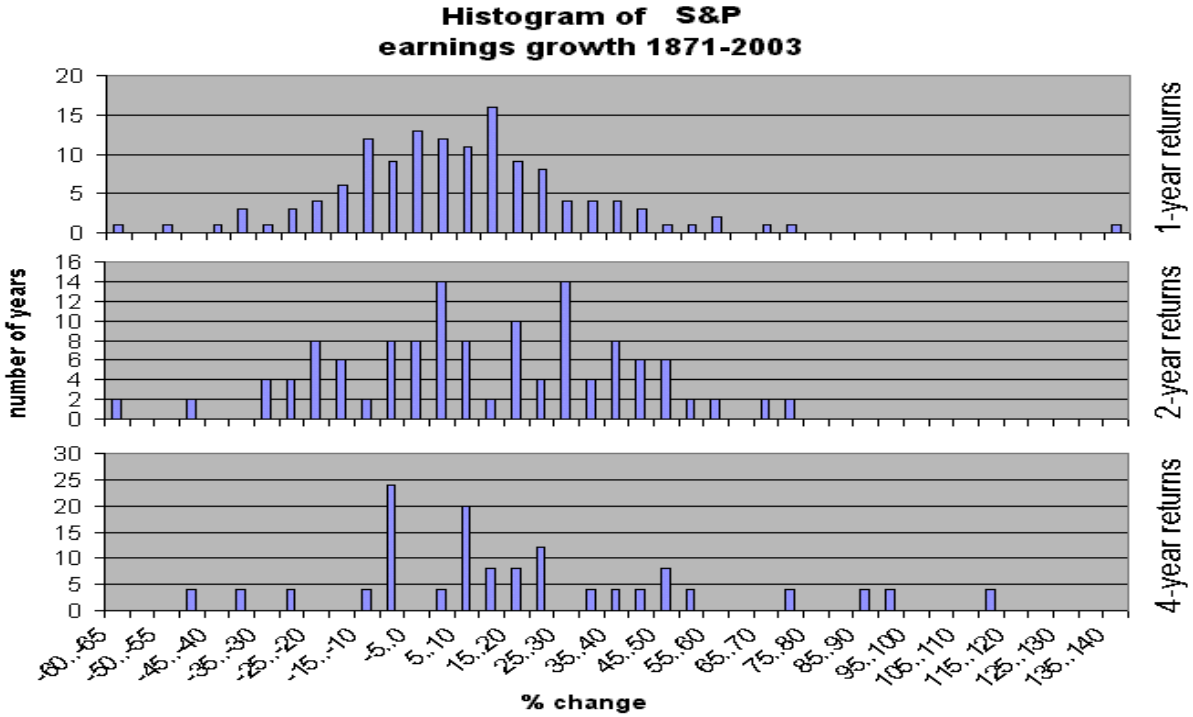

Figure 7. Distribution of S&P 500 1, 2 and 4-year Earnings Growth

The 1-year returns show a slight central valley, with multiple lobes developing on the negative side and extremely fat tails. The 2-year returns show an extreme central valley and multiple large lobes on both sides. The 4-year returns show just how erratic long-horizon returns can be for actual market data. The 8-year returns, not shown, are basically flat.

Butterfly distributions probably imply that actual markets are more non-linear than is apparent from the growth-binomial distributions analysis presented in the first section, i.e. that the median is even more depressed from the average. Additional investigation is planned using persistence (for example) to attempt to simulate butterfly distributions.

Note that butterfly distributions were observed in the probability distributions of the returns of a single investment, or the aggregate returns of a portfolio. They may not apply to the probability distribution of returns among a group of investments (or companies). The probable cause of butterfly distributions is the persistence of returns for several periods. When a company has good times, they probably last several years, and bad times probably last as well. This causes the probable returns to diverge from the average. Theoretical distributions are looking at alternate possible futures. Histograms such as those above are looking at different periods of time in the past. For example, the 2-year returns histogram looks at every distinct 2-year period for the past 132 years.

But when looking at how returns vary among companies, over a particular period of time, we wouldn't expect necessarily to find an equivalent persistence effect. We do however find excessive volatility, fat tails, and the sort of lopsided distribution with widely separate mean and median that we have noted earlier. Consider for example the figure below showing 10 year returns, by company, for the members of the NASDAQ 100 index. The left side is truncated at –100%, which is the worst possible return given the limited liability nature of equities. The right side has such extended tails that the scale had to be changed by an order of magnitude to get

them to fit on the page. If plotted on a constant scale, the last entry would be about three times the width of the page to the right.

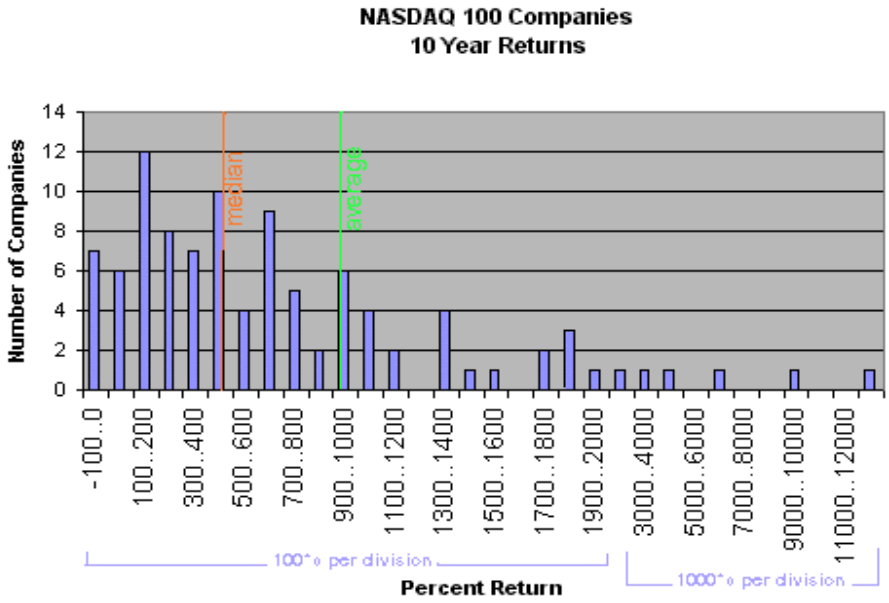

Figure 8. Distribution of Price Returns for NASDAQ 100 Companies, 1994-2003

There is some irregularity that might bear further investigation, but certainly nothing as clear as the central dip in the 2-year returns for the S&P aggregated return histogram.

Notice how much the median differs from the average. Since all the current members of the NASDAQ 100 were not in existence for 10 years, 4-year returns were used to compare mean and median to actual returns, as shown in the table below:

average	+56% (un-weighted)
median	+27% (un-weighted)
actual	-45% (cap-weighted)

Table 3. Average, Median & Actual Returns for NASDAQ 100 Companies, 1999-2003

It is not a perfect comparison, since the actual return of the NASDAQ 100 index is a cap-weighted value and the values inferred from our analysis are not weighted. But it is interesting to an actual market example in which the actual return is so much less than even the median. This same observation holds for Shiller's S&P 500 data.

Law of Control Conjecture. Lower returns will be associated with greater control. For example, cheaper (higher return) airline tickets will have restrictions on travel times and changes. A controlling partner will have to give higher returns to silent partners to compensate them for their lack of control. The law of control, as a conjecture, is at least consistent with higher returns for mutual and index funds, in which shareholders lose voting rights, even though we have earlier seen alternate explanations for this return differential.

Control is worth something, because it can be used to affect other interests of the investor exercising control. This could be anything from use of the corporate jet, to awarding favorable contracts to other companies owned by the investor, to sponsoring environmental or aesthetic interests of the investor. However, in public companies there may be regulations that inhibit

some uses of control. Use of control to affect general societal quality of life issues, like how much pollution the company produces, is hard to assign a monetary value.

The law of control, if true, has serious social implications when coupled with the revised law of highest returns. It implies that business activity evolves toward loss of control as it seeks higher return, and that ultimately business activity will be ungoverned.

Law of Disaster Conjecture. Even a tiny probability of zero return will eventually wipe out all portfolios. Markets appear to believe there is zero possibility of such a disaster. Analysis of this sort is probably better handled by the Crash Rate Theory already mentioned.

Asymmetric Returns Puzzle. Options, which include equity itself as described earlier, and limited liability companies, are legal instruments for shaping returns distributions. They are attempts at obtaining differentially higher returns by eliminating negative returns (losses exceeding the amount of capital allocated to the business). The elimination or reduction of one side of the returns distribution is an "asymmetry."

One would think that bondholders, suppliers, customers and all parties contracting with such a business would look unfavorably on the liability limitation, and require concessions and rates that would compensate for the increased risk of dealing with such an entity.

Such concessions should adversely affect the returns possible from an equity-optioned or limited liability business. Private businesses not employing these techniques should in theory enjoy higher returns in exchange for the higher risk they take. And asymmetric returns should be lower than symmetric ones. But this does not seem to happen. Something is wrong with this picture, thus the term "puzzle."

Because Crash Rate Theory relies on negative returns to make rational adjustments to the crash rate, the apparent lack of proper handling of asymmetric returns by the economy suggests the risk of disaster (crash rate) may be much higher than it should be.

The asymmetric returns puzzle might be related to the growth-yield conjecture. Lower interest rates afforded growth companies, possibly because they are considered more likely to repay, may correspond to favorable terms granted to limited liability companies. Even though the liability limitation is a negative, such companies may be judged more likely to succeed.

PORTFOLIO ADVICE

Many people ask if they can use non-linear portfolio theory to make money. The answer is yes, but it is complicated. Keep in mind that the main point of the work done so far has been to understand markets better, particularly to understand why people do not seem to make as much money by investing as financial professionals seem to think they will, to explain certain puzzles of interest to economists, and to point out the tendency for social damage inherent in some aspects of how markets are currently implemented. To optimize one's portfolio within the current market framework requires a certain amount of "joining the enemy," and may even involve investing in companies whose behavior you don't like.

Final portfolio advice will have to await a repeat analysis of non-linear market theory using a persistent returns distribution that gives simulated returns actually comparable to market returns. And the ideal portfolio would be a mutual fund run according to non-linear market theory principles, which doesn't exist yet. So, what we can say thus far is limited to the following.

Portfolio Size. It is almost certainly unsafe to hold fewer than 20 individual stocks. 40 would be better. More than 40 is probably too many for the average investor to research and keep up with. Fewer than 10 and you will probably lose money over a period of five years, regardless of what the market is doing. While Warren Buffet advocates a smaller, closely watched portfolio, most of us don't have the connections or insight that he does. We all know someone who bought one stock and it went up fabulously. But if you search among your extended circle of friends, you can find many equivalent stories of just plain luck. By the law of median returns, chances are if you buy one stock it is going down. Using non-linear statistical analysis is a way of coping with the poor quality of information about the future prospects of individual companies.

Portfolio Composition. In constructing a portfolio of individual stocks, pay attention to dividends, and have at least 4 or 5 uncorrelated types of stocks. Purchase equal dollar amounts of each stock. Attempting to buy more of one or the other will undermine portfolio diversification, and is just as faulty as trying to "time the market." Make sure that both small and large caps are represented, and that value, recovery and growth stocks are all represented. A minimum portfolio size to execute this strategy efficiently is about $200,000.

Small Portfolio Strategies. For smaller portfolios, the only sane option is to hold index funds. Managed mutual funds can be as volatile as individual stocks, because fund managers tend to introduce correlation either because of the fund's charter (e.g. sector funds) or because of the fund manager's style. Additionally, with a managed fund, you have no guarantee that the fund manager will maintain the composition of the portfolio. More often than not, they change it just when they should be sticking it out. Index fund composition is more stable, and the process for changing composition more formal.

Funds have the drawback that you give up voting rights, and cannot participate in corporate governance. One can slightly compensate for this by investing in the few funds that promise a certain style of investing, such as environmental funds.

Asset Allocation. Compute the annual (not daily) volatility of your portfolio. I am preparing a web script to assist with this, but for now you'll have to use Excel or a similar tool. Use at least 5 years worth of returns, expressed as G+Y (earnings growth plus dividend yield). Use 10 years if you can get it. Price is an acceptable way to figure returns for this purpose only for non-dividend paying stocks.

If the standard deviation of the annual composite returns is more than 15% of the base return, you need to allocate some of your portfolio to cash or similar risk free assets. Use the Effectiveness of Re-balancing chart in the first section to estimate your cash allocation, and add a 20% safety margin. For now, the only analysis on cash re-balancing has been for annual re-balancing. Pick a fixed month of the year to conduct your re-balancing.

For the time being, do not re-balance among the equity portion of your portfolio. There is no definite indication that it is helpful for standard returns distributions, and analysis of persistent distributions is incomplete.

Disclaimer. Note that the author is not a professional financial advisor, and manages only his own portfolio.

CONCLUSION

Summary of Non-Linear Market Theory. There are many other points in non-linear market theory, some of which need further investigation such as the unexpected inverse relation between interest rates and growth. But these are the main ones and the ones most solidly supported. And the exploration of the social implications could go on for a lifetime, you only have the start of it here.

In a nutshell, because of the nearer to average returns and larger N afforded by larger portfolios, the lucky rich get richer and pool their resources in large ungovernable pools that soon absorb the government.

At first masquerading as beneficiaries of humankind, these economic processes ultimately act to preserve the differential G-R and increase N at any cost. Statistically, they already deplete the resources of anyone who does not join the pool through the depression of median returns. It is a simple matter to increase volatility, depressing the median even further, and turning it negative. When markets become saturated, corporations seek to prolong the period of excess return by decreasing costs, which means decreasing the quality of life for their workers.

It is not a simple matter to oppose this infinite force. It appears from examining history and the G+Y-R chart (Figure 4) that efforts were made to regulate industry in the U.S. beginning in 1900 and accelerating in the 1930's. This backfired by temporarily raising the equity premium, and by mid-century an outcry against regulation and "inefficiency" had already begun, which reached full steam in the Reagan administration and continues to have momentum today. Other countries, e.g. Britain, attempted to socialize industry, or in the case of Russia completely control it. These attempts could last only until the pressure from the accumulation of slightly higher returns elsewhere overwhelmed the social institutions that opposed the law of equal returns driven by the growth equation.

But the market driven economy is proving to generate more dystopia than utopia, and ultimately must lead to some unfortunate collapse in the future, as indeed it did a century ago. Our logic here only predicts the rise, not the crash, and only predicts the difficulty of solving the problem, pointing out the naivety of social solutions tried in the past. Some glimpse of the dynamics that control the crashes and the methods that might be effective against them are the province of the Crash Rate Theory.

An Ethical & Spiritual Afterword. I feel I should address an attitude prevalent among some people of faith that God must certainly have ordained "all this" and is in control and will at the appropriate time "take care of it."

The role of whatever ethical or spiritual principles one subscribes to is, in my opinion, to inspire a notion of value, and to motivate humans to take some action to assert and preserve that value. In my reading of the New Testament, the central defining document of protestant Christianity for example, I interpret no promise of future direct divine intervention to solve problems we've created. Rather I read a story of an intervention that was already intended to be sufficient. If it were not sufficient, but only a promise, most Christians would consider their faith to have been misplaced. I therefore dismiss the so-called "fundamentalists," who think all of this is just leading up to some sort of Second Coming and worldly, utopia as being as irrelevant as the Branch Davidians, and as troublesome.

While humans may not be able to solve their own problems by their own efforts, clearly they are asked to make responsible moral and value choices, take stock of the ethical systems they have created, and make corrections as needed. This is exactly the example set by Jesus, Buddha, Mohammed and numerous other spiritual leaders.

Jesus took stock of the way the Hebrew Law was being applied, and said basically "You've got it all wrong, you should put people first, not the law or the institutions." The Buddha took stock of the caste system, which could after all be viewed as a structuring of access to economic returns, and said basically “You’ve got it all wrong, you should put people first, not the caste they were born into.” In a similar way, Mohammed took formerly enslaved and economically outcast people and placed them in high regard, much to the dismay of the Meccans.

In each case, as time passed and the faithful became commercially successful, and the process of median returns re-stratified them, sacrifices within the vision of freedom and dignity were made for the perpetuation of commercial success. Western Christianity and Middle Eastern Islam both fell back into the evil practice of slavery. Buddhism withdrew into its monasteries. The vision of each spiritual founder was eroded by the practical infinity of differential returns.

We need to take stock of ethical values like "private property rights" and the "rights of corporations" to see if they are really working. And in devising solutions, we need to take into account the mathematical laws by which the world is governed, some of which I have outlined here. These are, since they are part of creation merely discovered and not created by man, after all divine laws. Most economic thinking I find, unfortunately, to be wishful delusion created by overly optimistic humans. It derives from a sense of fair play evolved among primates living in small groups, and who had not yet invented infinity.

In other words, a spiritual or ethical man or woman recognizes that ethical decision making will not survive the law of equal (or highest) returns, which is an infinite force. Instead, a spiritual or ethical person seeks to eliminate inequities such as artificial personhood, artificial limitation of liability, customary usurpation of voting privileges by funding pools, and indeed all artificial and false promises of utopia, so that the spirit has the chance to take root or fail in the nature of each individual.